

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531***. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (3 marks each) Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A and B are square matrices of the same size and c is a scalar then $\text{tr}(A^T + cB) = \text{tr}(A) + c\text{tr}(B)$.

Question 3. (5 marks) Show that the reduced row echelon form of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if $ad - bc \neq 0$.

Question 2. (3 marks) Find all 2×2 matrices M such that $MA - AM = 0$ where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Question 3. (6 marks) Find the values, if any, of k for which the following system has:

$$\begin{cases} x + y + kz = 1 \\ x + ky + z = 1 \\ kx + y + z = 1 \end{cases}$$

Exactly one solution, no solutions, infinitely many solutions.

Bonus Question. (3 marks) If A , B and C are matrices such that the operations are defined, show that $A(BC) = (AB)C$.