Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2025: **Quiz 5** name: _____

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A and B are $n \times n$ matrices, if $AB = -BA$, and if n is odd, show that either A or B has no inverse.

Question 2. *(3 marks)* Let A and B be two 3×3 matrices such that $\det(A) = -2$ and $\det(B) = 3$. Find the following: $\det\left((-2B^3)^{-1}(3A^3)^T \operatorname{adj}(3A)\right)$.

Question 3. *(5 marks)* Let P be the point $(2,3,-2)$ and Q the point $(7,-4,1)$. Only using vectors, find the point on the line segment connecting the points P and Q that is $\frac{3}{4}$ of the way from P to Q.

Question 4.[1] Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$. Given: $||\vec{u}|| = 5$, $||\vec{u}+2\vec{v}|| = \sqrt{2}$, $\vec{v}$ and $\vec{u}+3\vec{v}$ are both unit vectors, and the angle between $\vec{u}+2\vec{v}$ and $\vec{u}+3\vec{v}$ is $\pi/4$.

a. *(3 marks)* Find $\vec{u}\cdot\vec{v}$.

b. *(2 marks)* Find $||\vec{u}+\vec{v}||$.

[1] From or modified from a John Abbott final examination