Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2025: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A and B are $n \times n$ matrices, if $AB = -BA$, and if n is odd, show that either A or B has no inverse.

True,

From premise $AB = -BA$

$$\det(AB) = \det(-BA)$$
$$\det(A)\det(B) = (-1)^n \det(B)\det(A)$$
$$\det(A)\det(B) = -\det(B)\det(A) \quad \text{since } n \text{ is odd}$$
$$\det(A)\det(B) = -\det(A)\det(B)$$
$$2\det(A)\det(B) = 0$$
$$\det(A)\det(B) = 0$$

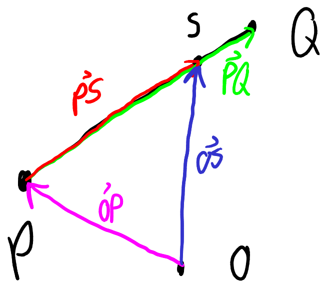
$\therefore$ $\det(A) = 0$ or $\det(B) = 0$

$\therefore$ by equivalence theorem A or B is singular.

Question 2. *(3 marks)* Let A and B be two 3×3 matrices such that $\det(A) = -2$ and $\det(B) = 3$. Find the following: $\det\left((-2B^3)^{-1}(3A^3)^T \operatorname{adj}(3A)\right)$.

$$= \det((-2B^3)^{-1}) \det((3A^3)^T \operatorname{adj}(3A))$$
$$= \frac{1}{\det(-2B^3)} \det(3A^3)^T \det(\operatorname{adj}(3A))$$
$$= \frac{1}{(-2)^3\det(B^3)} \det(3A^3)(\det(3A))^{3-1}$$
$$= \frac{1}{-8(\det B)^3} 3^3\det(A^3)(3^3\det A)^2$$
$$= \frac{1}{-8\,\cancel{3^3}} \cancel{3^3}(\det A)^3\, 3^6 (\det(A))^2$$
$$= \frac{1}{\cancel{-8}} \cancel{(-2)^3}\, 3^6(-2)^2$$
$$= 2^2 3^6$$

Question 3. *(5 marks)* Let P be the point $(2,3,-2)$ and Q the point $(7,-4,1)$. Only using vectors, find the point on the line segment connecting the points P and Q that is $\frac{3}{4}$ of the way from P to Q.



We have that $\vec{PS} = \frac{3}{4}\vec{PQ}$

$= \frac{3}{4}(\vec{OQ} - \vec{OP})$

$= \frac{3}{4}((7,-4,1) - (2,3,-2))$

$= \frac{3}{4}(5,-7,3)$

and $\vec{OS} = \vec{OP} + \vec{PS}$

$= (2,3,-2) + \frac{3}{4}(5,-7,3)$

$= \left(\frac{19}{4}, \frac{-9}{4}, \frac{1}{4}\right)$

$\therefore S\left(\frac{23}{4}, \frac{-9}{4}, \frac{1}{4}\right)$

Question 4.[1] Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$. Given: $||\vec{u}|| = 5$, $||\vec{u}+2\vec{v}|| = \sqrt{2}$, $\vec{v}$ and $\vec{u}+3\vec{v}$ are both unit vectors, and the angle between $\vec{u}+2\vec{v}$ and $\vec{u}+3\vec{v}$ is $\pi/4$.

a. *(3 marks)* Find $\vec{u}\cdot\vec{v}$.

b. *(2 marks)* Find $||\vec{u}+\vec{v}||$.

a)

$(\underline{u}+2\underline{v})\cdot(\underline{u}+3\underline{v}) = ||\underline{u}+2\underline{v}||\,||\underline{u}+3\underline{v}||\cos\frac{\pi}{4}$

$\underline{u}\cdot\underline{u} + \underline{u}\cdot(3\underline{v}) + (2\underline{v})\cdot\underline{u} + (2\underline{v})\cdot(3\underline{v}) = \sqrt{2}\cdot 1\,\frac{1}{\sqrt{2}}$

$||\underline{u}||^2 + 3\underline{u}\cdot\underline{v} + 2\underline{v}\cdot\underline{u} + 6\underline{v}\cdot\underline{v} = 1$

$5^2 + 5\underline{u}\cdot\underline{v} + 6||\underline{v}||^2 = 1$

$5\underline{u}\cdot\underline{v} = -24 - 6(1)^2$

$5\underline{u}\cdot\underline{v} = -30$

$\underline{u}\cdot\underline{v} = -6$

b) $||\underline{u}+\underline{v}||^2 = (\underline{u}+\underline{v})\cdot(\underline{u}+\underline{v})$

$= \underline{u}\cdot\underline{u} + \underline{v}\cdot\underline{u} + \underline{u}\cdot\underline{v} + \underline{v}\cdot\underline{v}$

$= ||\underline{u}||^2 + 2\underline{u}\cdot\underline{v} + ||\underline{v}||^2$

$= 5^2 + 2(-6) + 1^2$

$= 25 - 12 + 1$

$= 14$

$\therefore ||\underline{u}+\underline{v}|| = \sqrt{14}$

[1] From or modified from a John Abbott final examination