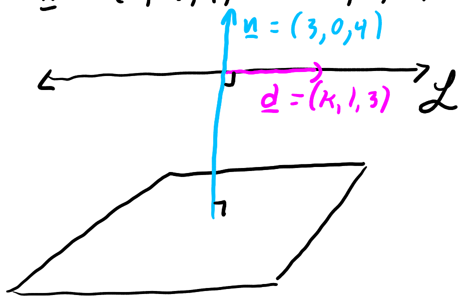
Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Consider the lines $\mathscr{L}: \begin{cases} x = kt + 7 \\ y = t - 3 \\ z = 3t + 4 \end{cases}$, $t \in \mathbb{R}$ and the plane $\mathscr{P}: 3x + 4z = 7$

a. *(3 marks)* Determine the values of k, if any, for which $\mathscr{L}$ is parallel to $\mathscr{P}$.

$\mathscr{L}: \underline{x} = (7,-3,4) + t(k,1,3)$



$\mathscr{L}$ is parallel to $\mathscr{P}$ if $\underline{n} \perp \underline{d}$, so if $n \cdot \underline{d} = 0$

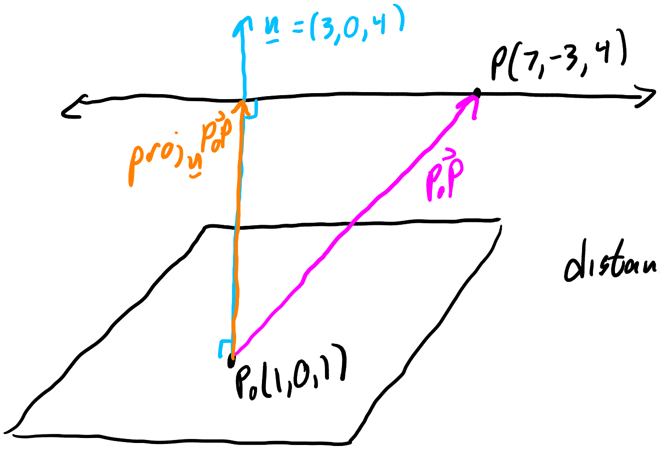
$0 = (3,0,4)\cdot(k,1,3)$

$0 = 3k + 12$

$k = -4$

$\therefore$ parallel if $k = -4$

b. *(5 marks)* If such a k value exists, find the distance from the line $\mathscr{L}$ to the plane $\mathscr{P}$ using projections.



$\mathscr{L}: \underline{x} = \underbrace{(7,-3,4)}_{\underline{P}} + t(-4,1,3)$

$\vec{P_0P} = \vec{OP} - \vec{OP_0}$

$= (7,-3,4) - (1,0,1)$

$= (6,-3,3)$

distance $= \|\text{proj}_{\underline{n}} \vec{P_0P}\|$

$= \left\| \dfrac{(6,-3,3)(3,0,4)}{(3,0,4)\cdot(3,0,4)}(3,0,4) \right\|$

$= \left\| \dfrac{18+12}{9+16}(3,0,4) \right\|$

$= \left\| \dfrac{30}{25}(3,0,4) \right\|$

$= \dfrac{6}{5} \|(3,0,4)\|$

$= \dfrac{6}{5}(5)$

$= 6$

Question 2. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

> If $\mathbf{x}_1$ and $\mathbf{x}_2$ are two solutions of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_2$ is a solution of the corresponding homogeneous linear system.

True,

$$\begin{aligned} A(\underset{\sim}{x}_1 - \underset{\sim}{x}_2) &= A\underset{\sim}{x}_1 - Ax_2 \\ &= \underset{\sim}{b} - \underset{\sim}{b} \\ &= \underset{\sim}{0} \end{aligned}$$

Question 3. *(2 marks)* Find the parametric equation of the plane $x + 2y + 3z = 4$.

Let $y = s$, $z = t$ $\quad s, t \in \mathbb{R}$

$$x + 2s + 3t = 4$$
$$x = 4 - 2s - 3t$$

$$\therefore \underset{\sim}{x} = (4-2s-3t, s, t) = (4,0,0) + s(-2,1,0) + t(-3,0,1) \qquad s, t \in \mathbb{R}$$

Question 4. *(3 marks)* Consider the system with equations: $x + y + z = b_1$, $x + 2y + cz = b_2$ and $x + 3y + dz = b_3$ where b_1, b_2, b_3, c, d are fixed real values, $P(1,1,1)$ satisfies all three equations and the solution set of the corresponding homogeneous linear system is $\mathbf{x} = t(2,-1,-1)$ where $t \in \mathbb{R}$.

Using a clearly labelled sketch give a geometrical interpretation of the linear system and its solution set, and the corresponding homogeneous linear system and its solution set.

We notice that the three equations are planes in $\mathbb{R}^3$ with normals

$\underset{\sim}{n}_1 = (1,1,1)$ The three planes have different inclinations since the normals are
$\underset{\sim}{n}_2 = (1,2,c)$ not multiples of each other
$\underset{\sim}{n}_3 = (1,3,d)$