Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S8: Winter 2025: **Quiz 8** name: ______________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Let V and W be subspaces of $\mathbb{R}^2$ that are spanned by $(3,\ 1)$ and $(2,\ 1)$, respectively. Find a vector $\mathbf{v}$ in V and a vector $\mathbf{w}$ in W for which $\mathbf{v}+\mathbf{w}=(3,\ 5)$.

Question 2. *(5 marks)* Prove that if $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ is linearly independent and $\mathbf{v}_3$ does not lie in span$\{\mathbf{v}_1,\ \mathbf{v}_2\}$, then $\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$ is linearly independent.

Question 3. Let W be the subspace of all polyomials in $\mathbb{P}_3$ such that $p(1) = 0$

a. *(4 marks)* Find a basis $\mathscr{B}$ of W.

b. *(1.5 marks)* State $\dim(\mathbb{P}_3)$, $\dim(W)$, and $\dim\left(\{0+0x+0x^2+0x^3\}\right)$.

c. *(1 mark)* Find the coordinate vector of $p(x) = 1+x+x^2-3x^3$ relative to the basis found in part a.

Question 4. *(3.5 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

Every basis of $\mathbb{P}_4$ contains at least one polynomial of degree 3 or less.

Bonus. *(1 mark)* State your favorite proof in Linear Algebra.