Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** *(1 mark each)* Integrate the following indefinite integrals:

a.

$$\int \frac{1}{x^{1/3}} \, dx = \int x^{-1/3} dx = \frac{x^{2/3}}{2/3} + C = \frac{3X^{2/3}}{2} + C$$

b.

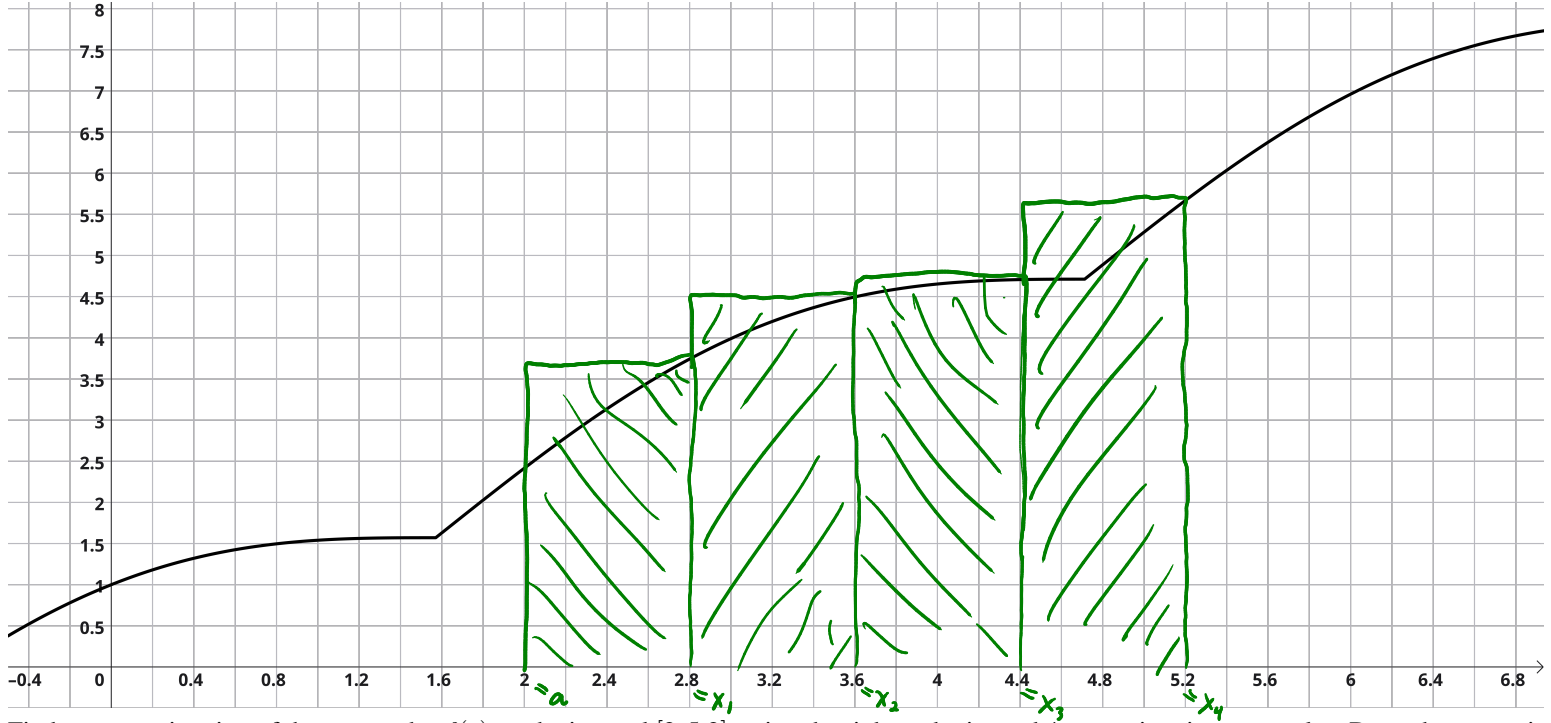$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

c.

$$\int \cot x \, dx = \ln|\sin x| + C$$

d.

$$\int \frac{1}{x\sqrt{5x^2 - 11}} \, dx = \int \frac{1}{x\sqrt{5\left(x^2 - \frac{11}{5}\right)}} dx$$

$$\rightarrow = \frac{1}{\sqrt{5}} \int \frac{1}{x\sqrt{x^2 - \left(\sqrt{\frac{11}{5}}\right)^2}} dx$$

$$= \frac{1}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{11}} \, \text{arcsec}\left(\frac{x}{\sqrt{\frac{11}{5}}}\right) + C$$

$$= \frac{1}{\sqrt{11}} \, \text{arcsec}\left(\frac{\sqrt{5}x}{\sqrt{11}}\right) + C$$

**Question 2.** *(5 marks)* The graph of $y = f(x) = x + |\cos x|$ is given below:



Find an approximation of the area under $f(x)$ on the interval $[2, 5.2]$, using the right endpoint and 4 approximating rectangles. Draw the approximating rectangles. **Use the function to find the approximation and not the graph**.

$n = 4$

$\Delta x = \frac{b-a}{n} = \frac{5.2-2}{4} = \frac{3.2}{4} = 0.8$

$x_i = a + i\Delta x = 2 + i(0.8)$

$x_1 = 2 + 1(0.8) = 2.8$
$x_2 = 2 + 2(0.8) = 3.6$
$x_3 = 2 + 3(0.8) = 4.4$
$x_4 \qquad = 5.2$

$\text{Area} \approx \sum_{i=1}^{n} \text{area of } R_i$

$= \sum_{i=1}^{n} f(x_i)\Delta x$

$= \Delta x \sum_{i=1}^{n} f(x_i)$

$= \Delta x \sum_{i=1}^{n} (x_i + |\cos(x_i)|)$

$= 0.8\left[2.8 + |\cos(2.8)|\right.$

$\qquad + 3.6 + |\cos(3.6)|$

$\qquad + 4.4 + |\cos(4.4)|$

$\qquad + \left.5.2 + |\cos(5.2)|\right]$

$\doteq 14.89$

**Question 3.** *(3 marks)* Determine whether the function $y = f(x)$ is a solution to the initial value problem (IVP) below. Show your work.

$$f(x) = \frac{x}{2} \arctan(2\ln x) - \frac{\pi}{4} x.$$

Determine whether $f(x)$ satisfies

$$y' = \arctan(2\ln x), \qquad y\left(\sqrt{e}\right) = 0,$$

for $x > 0$.

$$f'(x) = \underline{\arctan(2\ln x)} + \frac{x}{2} \cdot \frac{1}{1+(2\ln x)^2} \cdot \frac{2}{x} - \frac{\pi}{4}$$

$$\neq y'$$

$\therefore$ does not satisfy the differential equation.

$\therefore$ is not a solution.

$$y(\sqrt{e}) = f(\sqrt{e}) = \frac{\sqrt{e}}{2} \arctan(2\ln\sqrt{e}) - \frac{\pi}{4}\sqrt{e}$$

$$= \frac{\sqrt{e}}{2} \arctan\left(\frac{2}{2}\ln(e)\right) - \frac{\sqrt{e}\pi}{4}$$

$$= \frac{\sqrt{e}}{2} \arctan(1) - \sqrt{e}\frac{\pi}{4}$$

$$= \frac{\sqrt{e}}{8}\pi - \frac{\sqrt{e}}{4}\pi \neq 0$$

$\therefore$ does not satisfy the initial condition.

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**Question 4.** *(4 marks)* Sketch a region whose area is equal to $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \sqrt{1 - \left(\frac{i}{n}\right)^2}\frac{1}{n}$ and find the exact value of the area.
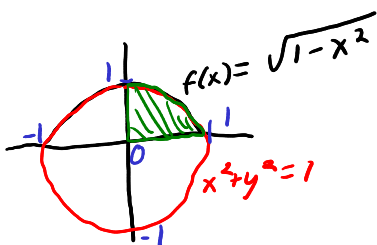
$$\Delta x = \frac{1}{n} = \frac{b-a}{n} \implies 1 = b-a$$

$$x_i = \frac{i}{n} = a + i\Delta x$$

$$\frac{i}{n} = a + \frac{i}{n} \implies a = 0$$

$$\implies b = 1$$

$$f(x) = \sqrt{1 - x^2}$$



$$f(x) = \sqrt{1-x^2}$$

$$x^2 + y^2 = 1$$

$$\text{Area} = \frac{\pi(1)^2}{4} = \frac{\pi}{4}$$