Dawson College: **Integral Calculus**: 201-SN3-RE-S2: Winter 2026: **Quiz 1** name: Y.Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Integrate the following indefinite integrals:

a.

$$\int \frac{1}{x^{1/5}}\,dx = \int x^{-1/5}\,dx = \frac{x^{4/5}}{4/5} = \frac{5x^{4/5}}{4} + C$$

b.

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

c.

$$\int \tan x\,dx = -\ln|\cos x| + C$$

d.

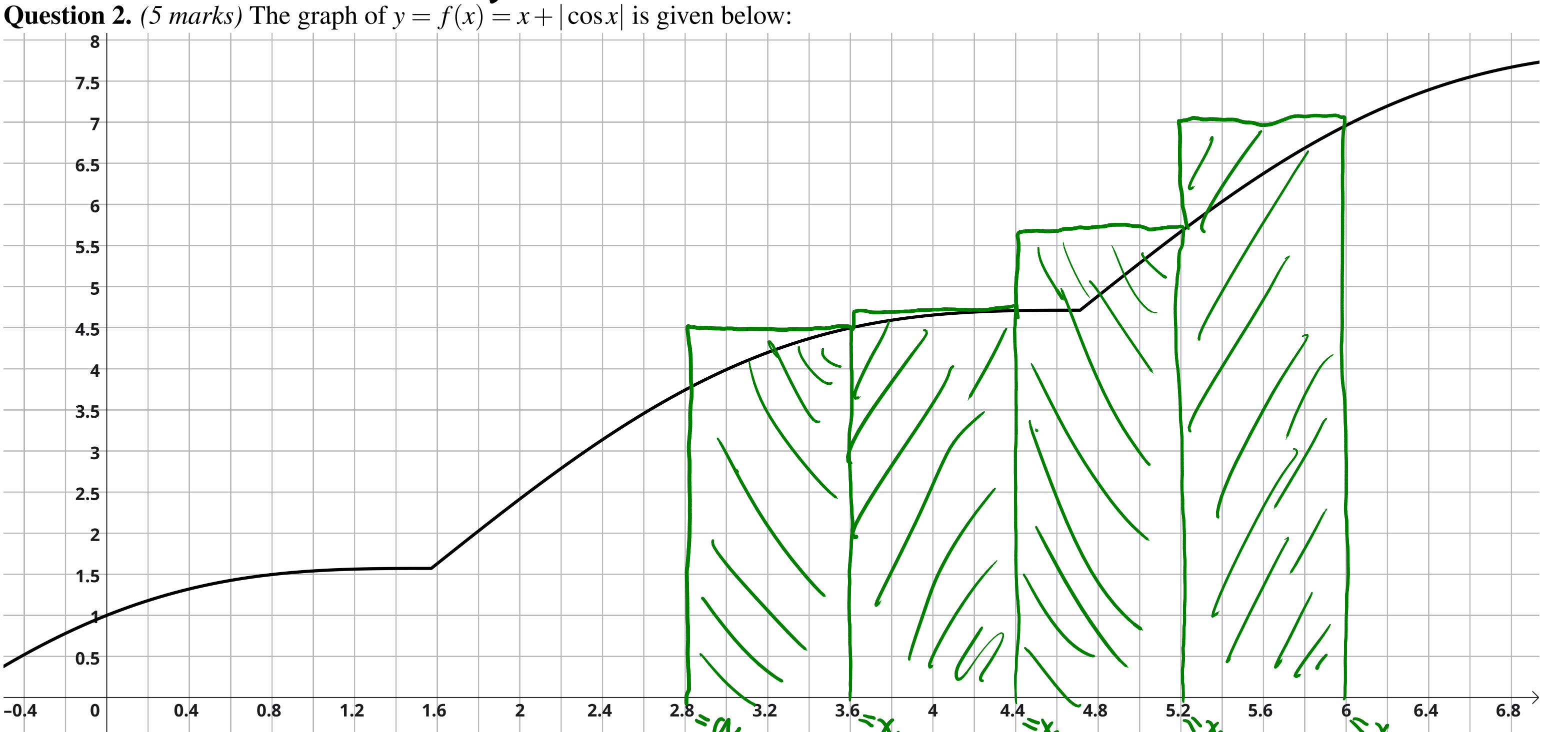
$$\int \frac{1}{\sqrt{13-5x^2}}\,dx = \int \frac{1}{\sqrt{5\left(\frac{13}{5}-x^2\right)}}\,dx$$

$$= \frac{1}{\sqrt{5}} \int \frac{1}{\sqrt{\left(\sqrt{\frac{13}{5}}\right)^2 - x^2}}\,dx$$

$$= \frac{1}{\sqrt{5}} \arcsin\left(\frac{x}{\sqrt{\frac{13}{5}}}\right) + C$$

$$= \frac{1}{\sqrt{5}} \arcsin\left(\frac{\sqrt{5}\,x}{\sqrt{13}}\right) + C$$

Question 2. *(5 marks)* The graph of $y = f(x) = x + |\cos x|$ is given below:



Find an approximation of the area under $f(x)$ on the interval $[2.8, 6]$, using the right endpoint and 4 approximating rectangles. Draw the approximating rectangles. **Use the function to find the approximation and not the graph**.

$n = 4$

$\Delta x = \frac{b-a}{n} = \frac{6-2.8}{4} = \frac{3.2}{4} = 0.8$

$x_i = a + i\Delta x = 2.8 + i(0.8)$

$x_1 = 2.8 + 1(0.8) = 3.6$

$x_2 = 2.8 + 2(0.8) = 4.4$

$x_3 = 2.8 + 3(0.8) = 5.2$

$x_4 = 6$

$$\text{Area} \approx \sum_{i=1}^{n} \text{area of } R_i$$

$$= \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \Delta x \sum_{i=1}^{n} f(x_i)$$

$$= \Delta x \sum_{i=1}^{n} \left(x_i + |\cos(x_i)|\right)$$

$$= 0.8\left[3.6 + |\cos(3.6)| + 4.4 + |\cos(4.4)| + 5.2 + |\cos(5.2)| + 6 + |\cos(6)|\right]$$

$$\doteq 17.47$$

Question 3. *(3 marks)* Determine whether the function $y = f(x)$ is a solution to the initial value problem (IVP) below. Show your work.

$$f(x) = \frac{\arctan(2\ln x) - \frac{\pi}{2}}{\ln x}.$$

Determine whether $f(x)$ satisfies

$$y' = \arctan(2\ln x), \qquad y(\sqrt{e}) = 0,$$

for $x > 0$, $x \neq 1$.

$$f'(x) = \frac{\left(\frac{1}{1+(2\ln x)^2} \cdot \frac{2}{x}\right)\ln x - \frac{1}{x}\left(\arctan(2\ln x) - \frac{\pi}{2}\right)}{(\ln x)^2}$$

$\neq y'$

$\therefore$ does not satisfy the differential equation.

$\therefore$ is not a solution.

$$y(\sqrt{e}) = \frac{\arctan(2\ln\sqrt{e}) - \frac{\pi}{2}}{\ln\sqrt{e}}$$

$$= \frac{\arctan\left(\frac{2}{2}\ln e\right) - \frac{\pi}{2}}{\frac{1}{2}\ln e}$$

$$= \frac{\arctan(1) - \frac{\pi}{2}}{1/2}$$

$$= \frac{\pi/4 - \pi/2}{1/2} \neq 0$$

$\therefore$ does not satisfy the initial condition.

Question 4. *(4 marks)* Sketch a region whose area is equal to $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 - \left(\frac{i}{n}\right)^2}\frac{1}{n}$ and find the exact value of the area.

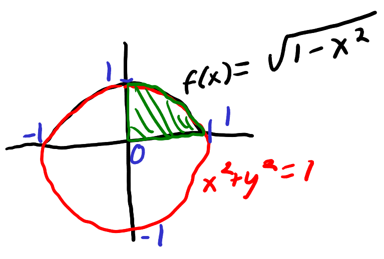
$\Delta x = \frac{1}{n} = \frac{b-a}{n} \Rightarrow 1 = b - a$

$x_i = \frac{i}{n} = a + i\Delta x$

$\frac{i}{n} = a + \frac{i}{n} \Rightarrow a = 0$

$\Rightarrow b = 1$

$f(x) = \sqrt{1 - x^2}$



$\text{Area} = \frac{\pi(1)^2}{4} = \frac{\pi}{4}$