Dawson College: **Integral Calculus**: 201-SN3-RE-S2: Winter 2026: **Quiz 2** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Formulae: $\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. Linear Algebra II __must__ be extremely fun.

b. Suppose $f(x)$ is integrable then $f(x)$ __might__ be continuous.

c. $\int_a^b f(x)\,dx$ __must__ be equal to $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ where $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$ if $f(x)$ is integrable.

d. $\int_a^b f(x)\,dx$ __must__ be equal to $\int_b^a -f(\alpha)\,d\alpha$ if $f(x)$ is integrable.

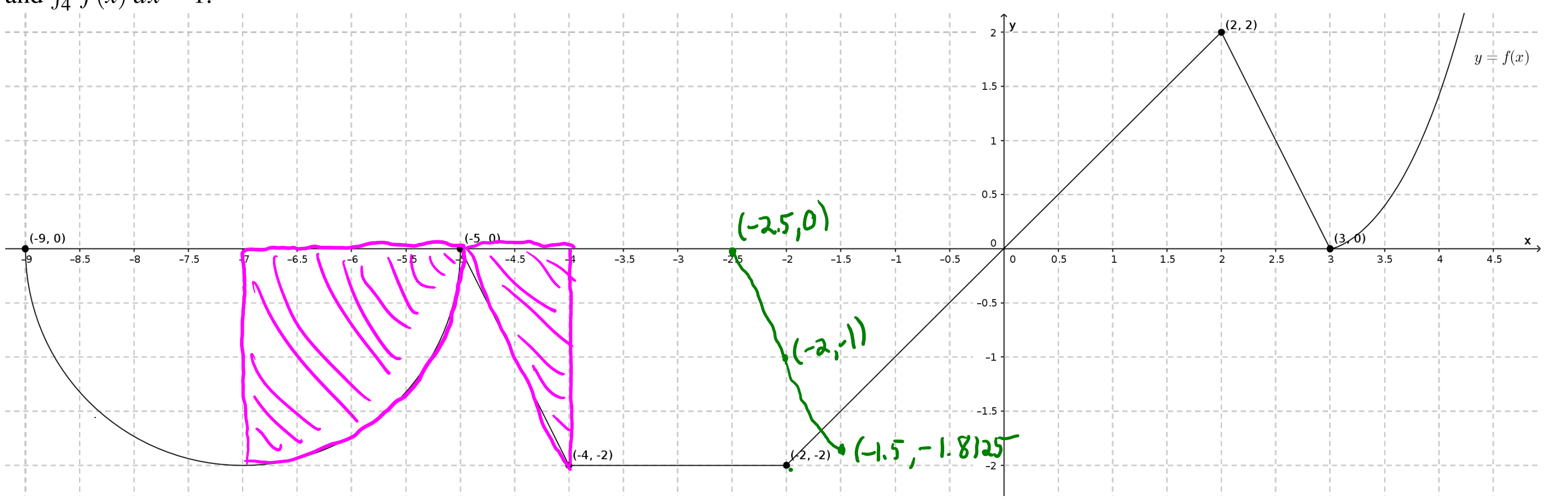
Question 2. *(2 marks)* Suppose $f(x)$ is integrable on $[a,\ b]$ and $c \in \mathbb{R}$. Prove $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$.

$$c\int_a^b f(x)dx = c\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} c\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} cf(x_i)\Delta x = \int_a^b cf(x)dx$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

since f(x) is integrable

Question 2. The graph of $y = f(x)$ consists of straight lines, one semicircle and a curve on the interval $[3,\infty)$. In addition, $\int_4^3 9f(x)\,dx + 4 = 0$ and $\int_4^5 f(x)\,dx = 1$.



a. *(3 marks)* Evaluate $\int_3^5 f(x)\,dx$. $= \int_3^4 f(x)dx + \int_4^5 f(x)dx = \frac{4}{9} + 1 = \frac{13}{9}$

$$\int_4^3 9f(x)dx = -4$$
$$9\int_4^3 f(x)dx = -4$$
$$-9\int_3^4 f(x)dx = -4$$
$$\int_3^4 f(x)dx = \frac{4}{9}$$

b. *(5 marks)* Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n}(i/n + f(x_i))\Delta x$ where $x_i = -7 + i\Delta x$ and $\Delta x = 3/n$.

$= a + i\Delta x$

c. *(2 marks)* Sketch the function $g(x) = \int_{-2.5}^{x} f(t)\,dt$ on $[-2.5,\ -1.5]$, on the above graph, and label 3 key points.

$$\Delta x = \frac{3}{n} = \frac{b-a}{n} \text{ where } a = -7 \Rightarrow b = -4$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{i\Delta x}{n} + f(x_i)\Delta x\right]$$

$$= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{i\Delta x}{n} + \sum_{i=1}^{n} f(x_i)\Delta x\right]$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{i\Delta x}{n} + \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty}\frac{\Delta x}{n}\sum_{i=1}^{n} i + \int_{-7}^{-4} f(x)dx$$

$$= \lim_{n\to\infty}\frac{3}{n^2}\frac{n(n+1)}{2} - \frac{\pi(2)^2}{4} - \frac{(2)(1)}{2}$$

$$= -\pi - 1 + \lim_{n\to\infty}\frac{3}{n}\frac{(n+1)}{2}$$

$$= -\pi + \frac{1}{2}$$

Question 3. *(5 marks)* Find the average value of the function $f(x) = \frac{\sin x + \sin x \tan^2 x}{\sec^2 x}$ on $[-\pi/3, \pi/4]$.

$$\text{Avg. val. of func.} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi/4 - (-\pi/3)} \int_{-\frac{\pi}{3}}^{\pi/4} \frac{\sin x + \sin x \tan^2 x}{\sec^2 x}\,dx$$

$$= \frac{1}{\frac{3\pi}{12} + \frac{4\pi}{12}} \int_{-\frac{\pi}{3}}^{\pi/4} \frac{\sin x\,(1+\tan^2 x)}{\sec^2 x}\,dx$$

$$= \frac{12}{7\pi} \int_{-\frac{\pi}{3}}^{\pi/4} \frac{\sin x \cancel{\sec^2 x}}{\cancel{\sec^2 x}}\,dx$$

$$= \frac{12}{7\pi}\Big[-\cos x\Big]_{-\frac{\pi}{3}}^{\pi/4}$$

$$= \frac{12}{7\pi}\left[-\cos\frac{\pi}{4} + \cos\left(\frac{-\pi}{3}\right)\right]$$

$$= \frac{12}{7\pi}\left[-\frac{1}{\sqrt{2}} + \frac{1}{2}\right]$$

$$= \frac{12}{7\pi}\left[\frac{\sqrt{2}-2}{2\sqrt{2}}\right]$$

$$= \frac{(\sqrt{2}-2)6}{7\pi\sqrt{2}}$$

Question 4. *(5 marks)* Given the function $f(x) = \int_{\arctan x}^{\pi/4} \frac{\tan(\tan t)}{\tan t}\,dt$. Find $f'(x)$ using the 2nd FTC presented in class and simplify completely. **(Show all your work!)**

$$= -\int_{\frac{\pi}{4}}^{\arctan x} \frac{\tan(\tan t)}{\tan t}\,dt$$

$$= -\int_{\frac{\pi}{4}}^{g(x)} \frac{\tan(\tan t)}{\tan t}\,dt \quad \text{where } g(x) = \arctan x \text{ and } g'(x) = \frac{1}{x^2+1}$$

$$= -h(g(x)) \quad \text{where } h(x) = \int_{\frac{\pi}{4}}^{x} \frac{\tan(\tan t)}{\tan t}\,dt$$

$$h'(x) = \frac{\tan(\tan x)}{\tan x} \quad \text{by 2nd FTC}$$

$$f'(x) = -h'(g(x))g'(x)$$

$$= -\frac{h'(\arctan x)}{x^2+1}$$

$$= -\frac{\tan(\tan(\arctan x))}{\tan(\arctan x)(x^2+1)} = \frac{-\tan x}{x(x^2+1)}$$

Bonus Question. *(2 marks)* Using the definition of the derivative of a function $g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$ simplify $g'(x)$ into a single term where $g(x) = \int_a^x f(t)\,dt$