Dawson College: **Integral Calculus**: 201-SN3-RE-S2: Winter 2026: **Quiz 3** name: Y. Lamoutagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Evaluate the following integral

$$\int \frac{e^{2x}+e^x+e^x\sec(\ln(e^x+1))}{e^x+1}\,dx$$

$$= \int \frac{e^{2x}+e^x}{e^x+1}\,dx + \int \frac{e^x \sec(\ln(e^x+1))}{e^x+1}\,dx$$

$u = \ln(e^x+1)$

$du = \frac{1}{e^x+1}\cdot e^x\,dx$

$$= \int \frac{e^x\cancel{(e^x+1)}}{\cancel{(e^x+1)}} + \int \sec u\,du$$

$$= e^x + \ln|\sec u + \tan u|$$

$$= e^x + \ln|\sec u + \tan u| + C$$

$$= e^x + \ln|\sec(\ln(e^x+1)) + \tan(\ln(e^x+1))| + C$$

Question 2. *(5 marks)* If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(-x)\,dx = \int_{-b}^{-a} f(x)\,dx$$

For the case where $f(x) \geq 0$ and $0 < a < b$, draw a diagram to interpret this equation geometrically as an equality of areas.

$$\int_a^b f(-x)\,dx = \int_{-a}^{-b} f(u)(-du) = \int_{-b}^{-a} f(u)\,du = \int_{-b}^{-a} f(x)\,dx$$

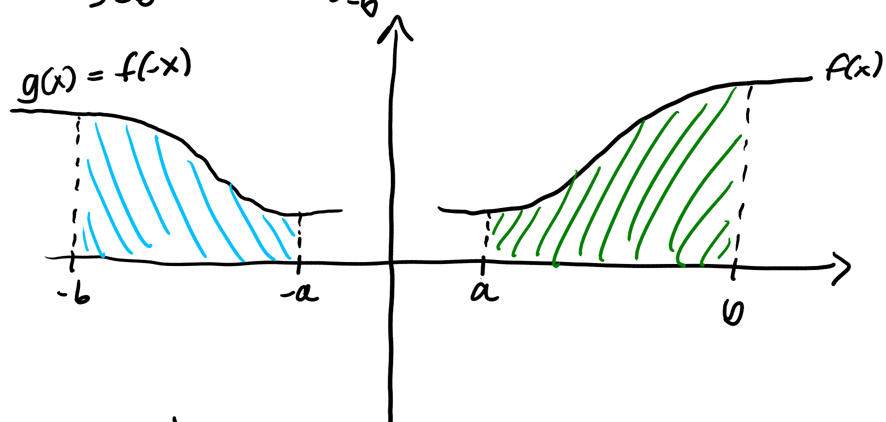
$u = -x$

$du = -dx$

$-du = dx$

$u(b) = -b$

$u(a) = -a$



Equal areas.

Question 3. *(5 marks)* Evaluate the following integral

$$\int_0^{1/3} \frac{1}{(9x^2+1)^{5/2}}\,dx$$

$$= \int_0^{1/3} \frac{1}{((3x)^2+1)^{5/2}}\,dx$$

$$= \int_0^{\pi/4} \frac{1}{(\tan^2\theta+1)^{5/2}}\,\frac{\sec^2\theta\,d\theta}{3}$$

$3x = \tan\theta$

$3dx = \sec^2\theta\,d\theta$

$3\frac{1}{3} = \tan\theta \Rightarrow \theta = \frac{\pi}{4}$

$3(0) = \tan\theta \Rightarrow \theta = 0$

$$= \frac{1}{3}\int_0^{\pi/4} \frac{1}{((\sec^2\theta)^{1/2})^5}\sec^2\theta\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} \frac{1}{(|\sec\theta|)^5}\sec^2\theta\,d\theta$$

$\sec\theta > 0$ on $[0, \frac{\pi}{4}]$

$$= \frac{1}{3}\int_0^{\pi/4} \frac{1}{\sec^5\theta}\sec^2\theta\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} \frac{1}{\sec^3\theta}\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} \cos^3\theta\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} \cos^2\theta\cos\theta\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} (1-\sin^2\theta)\cos\theta\,d\theta$$

$$= \frac{1}{3}\int_0^{1/\sqrt{2}} (1-u^2)\,du$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$u(0) = \sin(0) = 0$

$u(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$$= \frac{1}{3}\left[u - \frac{u^3}{3}\right]_0^{1/\sqrt{2}}$$

$$= \frac{1}{3}\left[\frac{1}{\sqrt{2}} - \frac{1}{3(\sqrt{2})^3}\right]$$

$$= \frac{5}{18\sqrt{2}}$$

Question 4. *(5 marks)* Evaluate the following integral

$$\int_{-\pi/12}^{\pi/12} \left(\frac{(x^2+1)\sin x}{x^4+1} + \cos^2 2x \right) dx$$

$= \int_{-\pi/12}^{\pi/12} f(x)dx + \int_{-\pi/12}^{\pi/12} \cos^2(2x)dx$

Let $f(x) = \frac{(x^2+1)\sin x}{x^4+1}$

$f(-x) = \frac{((-x)^2+1)\sin(-x)}{(-x)^4+1}$

$= \frac{(x^2+1)(-\sin x)}{x^4+1}$

$= -f(x)$

$\therefore f(x)$ is odd

since $f(x)$ is odd

$= 0 + \int_{-\pi/12}^{\pi/12} \frac{1+\cos 4x}{2} dx$

$= \frac{1}{2}\left[x + \frac{\sin 4x}{4}\right]_{-\frac{\pi}{12}}^{\pi/12}$

$= \frac{1}{2}\left[\left[\frac{\pi}{12} + \frac{\sin(\frac{\pi}{3})}{4}\right] - \left[-\frac{\pi}{12} + \frac{\sin(-\frac{\pi}{3})}{4}\right]\right]$

$= \frac{1}{2}\left[\frac{2\pi}{12} + \frac{2\sqrt{3}}{8}\right]$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{8}$

Question 5. *(5 marks)* Evaluate the following integral

$\int \frac{\ln x}{x^{3/2}} dx = uv - \int v du$

$u = \ln x \qquad du = \frac{1}{x}dx$

$v = -2x^{-1/2} \qquad dv = x^{-3/2}dx$

$= -2x^{-1/2}\ln x - \int -2x^{-1/2}\frac{1}{x}dx$

$= -\frac{2\ln x}{\sqrt{x}} + 2\int x^{-3/2}dx$

$= -\frac{2\ln x}{\sqrt{x}} + 2x^{-1/2}(-2) + C$

$= -\frac{2\ln x}{\sqrt{x}} - \frac{4}{\sqrt{x}} + C$

Question 6.

a. *(2 marks)* Use integration by parts to show that

$$\int f(x)\, dx = xf(x) - \int xf'(x)\, dx$$

$= uv - \int v du$

$u = f(x) \qquad du = f'(x)dx$

$v = x \qquad dv = dx$

b. *(3 marks)* If f and g are inverse functions and $f'(x)$ is continuous, prove that

$$\int_a^b f(x)\, dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)\, dy$$

(Hint: Use part (a.) and make the substitution $y = f(x)$.)

From a) $\int_a^b f(x)dx = \left[xf(x)\right]_a^b - \int_a^b xf'(x)dx$

$= bf(b) - af(a) - \int_a^b g(f(x))f'(x)dx$ since $f(x)$ and $g(x)$ are inv. func.

$y = f(x) \qquad du = f'(x)dx$

$y(b) = f(b)$

$y(a) = f(a)$

$= bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)dy$