Dawson College: **Integral Calculus**: 201-SN3-RE-S2: Winter 2026: **Quiz 3** name: ____________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Solve the differential equation

$$\frac{dx}{d\theta} = \frac{e^{2x}\sec^3(5\theta)\tan^3(5\theta)}{x}$$

$$\frac{x}{e^{2x}}dx = \sec^3(5\theta)\tan^3(5\theta)d\theta$$

$$\int xe^{-2x}\,dx = \int \sec^2(5\theta)\tan^2(5\theta)\sec(5\theta)\tan(5\theta)d\theta$$

$$uv - \int v\,du = \int \sec^2(5\theta)(\sec^2(5\theta)-1)\sec(5\theta)\tan(5\theta)d\theta$$

$u = x$ $du = dx$

$v = \frac{-e^{-2x}}{2}$ $dv = e^{-2x}dx$

$u = \sec(5\theta)$

$du = \sec(5\theta)\tan(5\theta)d\theta$

$\frac{du}{5} = \sec(5\theta)\tan(5\theta)d\theta$

$$-\frac{xe^{-2x}}{2} - \int -\frac{e^{-2x}}{2}dx = \int u^2(u^2-1)\frac{du}{5}$$

$$-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} = \frac{1}{5}\int u^4 - u^2\,du$$

$$-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} = \frac{1}{5}\left[\frac{u^5}{5} - \frac{u^3}{3}\right] + C$$

$$-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} = \frac{\sec^5(5\theta)}{25} - \frac{\sec^3(5\theta)}{15} + C$$

Question 2. *(5 marks)* Find the limit.

$$\lim_{x\to a^+} \cos x \frac{\int_0^{x-a}\ln t\,dt}{\int_a^x \ln(e^t - e^a)\,dt}$$

note: $\lim_{x\to a^+} \cos x = \cos a$

$$\lim_{x\to a^+} \frac{\int_0^{x-a}\ln t\,dt}{\int_a^x \ln(e^t-e^a)dt} \quad \text{I.F. } \frac{0}{0}$$

$$\overset{\hat{H}}{=} \lim_{x\to a^+} \frac{\ln(x-a)}{\ln(e^x-e^a)} \quad \text{I.F. } \frac{0}{0}$$

$$\overset{\hat{H}}{=} \lim_{x\to a^+} \frac{\frac{1}{x-a}}{\frac{1}{e^x-e^a}e^x}$$

$$= \lim_{x\to a^+} \frac{e^x-e^a}{e^x(x-a)} \quad \text{I.F. } \frac{0}{0}$$

$$\overset{\hat{H}}{=} \lim_{x\to a^+} \frac{e^x}{e^x + e^x(x-a)}$$

$$= \frac{e^a}{e^a} = 1$$

$$\therefore \lim_{x\to a^+} \cos x \frac{\int_0^{x-a}\ln t\,dt}{\int_a^x \ln(e^x-e^a)dt} = \lim_{x\to a^+}\cos x \lim_{x\to a^+} \frac{\int_0^{x-a}\ln t\,dt}{\int_0^x \ln(e^x-e^a)dt}$$

$$= \cos a \cdot 1$$

$$= \cos a$$

Question 3. *(5 marks)* Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx = \int_{-\infty}^{0} \frac{x^2}{9+x^6}dx + \int_0^{\infty} \frac{x^2}{9+x^6}dx$$

$$= \lim_{a\to-\infty} \int_a^0 \frac{x^2}{9+(x^3)^2}dx + \lim_{b\to\infty} \int_0^b \frac{x^2}{9+(x^3)^2}dx$$

$u = x^3$ $\quad$ $du = 3x^2dx$ $\quad$ $\frac{du}{3} = x^2dx$

$u(b) = b^3$ $\quad$ $u(0) = 0^3 = 0$ $\quad$ $u(a) = a^3$

$$= \lim_{a\to-\infty} \int_{a^3}^0 \frac{1}{9+u^2}\frac{du}{3} + \lim_{b\to\infty} \int_0^{b^3} \frac{1}{9+u^2}\frac{du}{3}$$

$$= \lim_{a\to-\infty} \left[\frac{1}{9}\arctan\frac{u}{3}\right]_{a^3}^0 + \lim_{b\to\infty} \left[\frac{1}{9}\arctan\frac{u}{3}\right]_0^{b^3}$$

$$= \lim_{a\to-\infty} \left[\frac{1}{9}\arctan 0 - \frac{1}{9}\underbrace{\arctan\frac{a^3}{3}}_{\to -\frac{\pi}{2}}\right] + \lim_{b\to\infty}\left[\frac{1}{9}\underbrace{\arctan\frac{b^3}{3}}_{\to \frac{\pi}{2}} - \frac{1}{9}\arctan 0\right]$$

$$= \frac{\pi}{9}$$

Question 4. *(5 marks)* Find the values of p for which the integral converges and evaluate the integral for those values of p.

$$\int_1^e \frac{1}{x(\ln x)^p}\,dx$$ infinite discontinuity at $x = 1$

$$= \lim_{a\to1^+} \int_a^e \frac{1}{x(\ln x)^p}dx$$

$$= \lim_{a\to1^+} \int_a^e \frac{1}{x(\ln x)^p}dx$$

$u = \ln x$ $\quad$ $du = \frac{1}{x}dx$ $\quad$ $u(e) = \ln e = 1$ $\quad$ $u(a) = \ln a$

$$= \lim_{a\to1^+} \int_{\ln a}^1 \frac{1}{u^p}du$$

if $p = 1$

$$= \lim_{a\to1^+} \Big[\ln|u|\Big]_{\ln a}^1 = \lim_{a\to1^+} \Big[\ln 1 - \ln|\ln a|\Big]$$ diverges to ∞

If $p \neq 1$

$$= \lim_{a\to1^+} \left[\frac{u^{-p+1}}{-p+1}\right]_{\ln a}^1 = \lim_{a\to1^+} \left[\frac{1}{1-p} - \frac{(\ln a)^{1-p}}{1-p}\right]$$

since $\ln a \to 0^+$ as $a \to 0^+$

if $1-p > 0$

$1 > p$ the integral converges to $\frac{1}{1-p}$

If $1 < p$ the integral diverges to ∞.