Dawson College: **Integral Calculus**: 201-SN3-RE-S2: Winter 2026: **Quiz 5** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Consider the region(s) $\mathscr{R}$ bounded by the following: $y = x^2 - x - 1$, $y - x = 2$, $x = 4$. Sketch the region $\mathscr{R}$ using intercept(s) and vertice(s), if any. Also clearly label the sketch. Set up, **but do not evaluate**, an integral or integrals that represents the area of the region $\mathscr{R}$.

Intersection between line and parabola: $x^2 - x - 1 = x + 2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

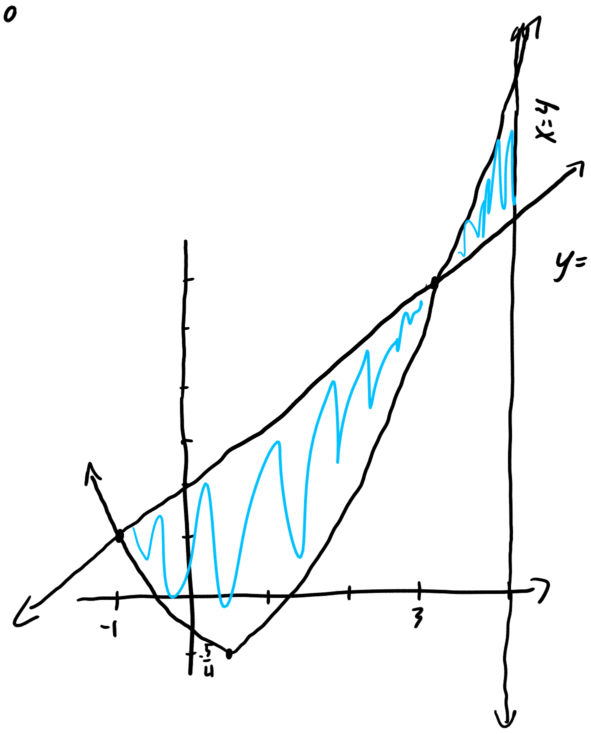
$x = 3 \quad x = -1$

vertex of parabola $y = x^2 - x - 1$

$= x^2 - x + \frac{1}{4} - \frac{1}{4} - 1$

$= (x - \frac{1}{2})^2 - \frac{5}{4}$

$$\text{Area} = \int_{-1}^{3} x + 2 - [x^2 - x - 1]\,dx + \int_{3}^{4} x^2 - x - 1 - [x+2]\,dx$$



Question 2. *(5 marks)* Find the values of c such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576. Sketch the region(s) using intercept(s) and vertice(s), if any. Also clearly label the sketch.

Intersection between the two curves $x^2 - c^2 = c^2 - x^2$

$2x^2 = 2c^2$

$x^2 = c^2$

$x = \pm c$

$y = x^2 - c^2$: x-int: $x = \pm c$

y-int = vertex: $(0, -c^2)$

$y = x^2 + c^2$:

x-int: $x = \pm c$

y-int = vertex: $(0, c^2)$

$$576 = \int_{-c}^{c} (c^2 - x^2) - (x^2 - c^2)\,dx$$

$$576 = \int_{-c}^{c} 2c^2 - 2x^2\,dx$$

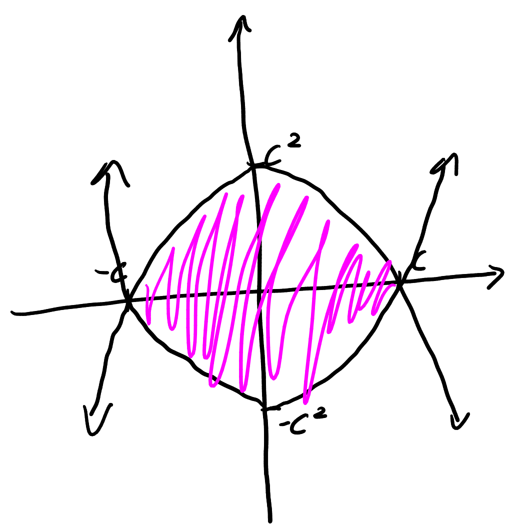
$$576 = \left[2c^2 x - \frac{2x^3}{3}\right]_{-c}^{c}$$

$$576 = \left(2c^2 c - \frac{2c^3}{3}\right) - \left(2c^2(-c) - \frac{2(-c)^3}{3}\right)$$

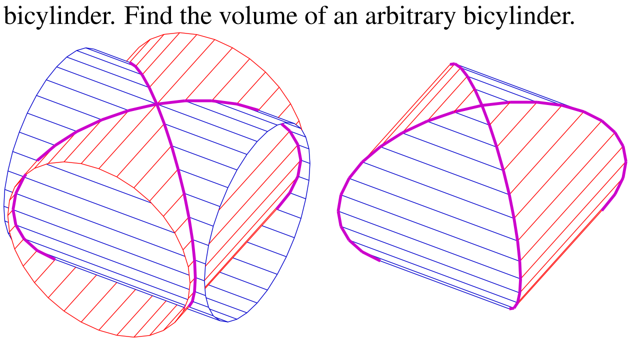
$$576 = 4c^3 - \frac{4}{3}c^3$$

$$576 = \frac{8}{3}c^3$$

$$c = 6$$

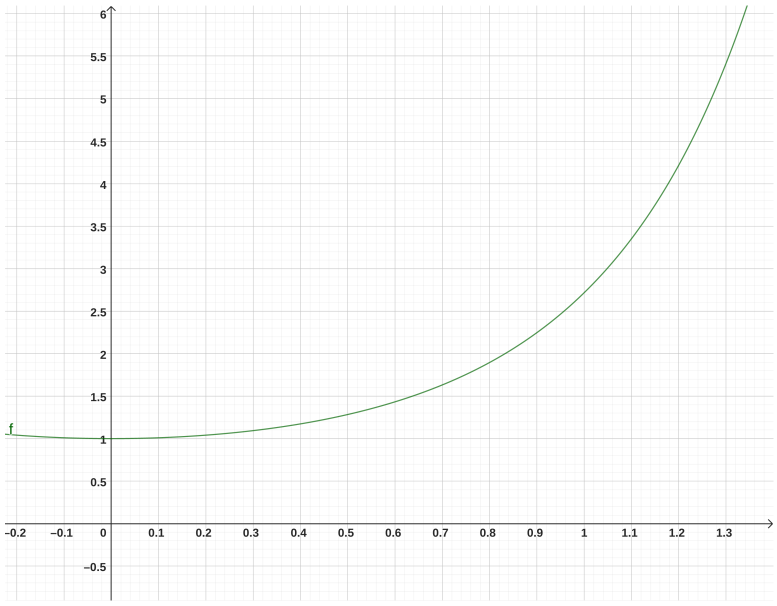


Bonus Question.[1] *(5 marks)(3 marks)* In geometry, a Steinmetz solid is the solid body obtained as the intersection of two or three cylinders of equal radius at right angles. Each of the curves of the intersection of two cylinders is an ellipse. The intersection of two cylinders is called a bicylinder. Find the volume of an arbitrary bicylinder.

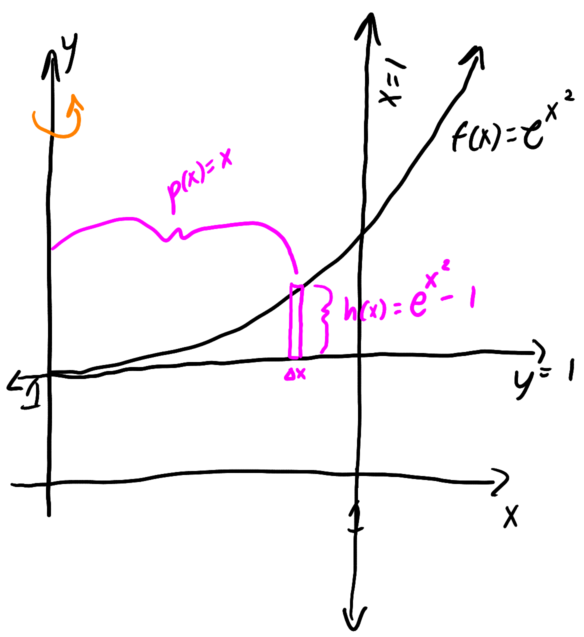


[1] https://en.wikipedia.org/wiki/Steinmetz_solid

Question 3. *(6 marks)*
Given the graph of $f(x) = e^{x^2}$.



Consider the region in the first quadrant $\mathscr{R}$ bounded by $y = e^{x^2}$, $x = 1$ and $y = 1$. Use the cylindrical shell method to find the volume of the solid obtained by revolving the region $\mathscr{R}$ about the y-axis. Sketch the region, draw a representative rectangle, write a representative element and label the sketch completely.



Volume of a single shell: $\Delta V = 2\pi p(x) h(x) \Delta x$

$$\text{Volume} = \lim_{n \to \infty} \sum_{i=1}^{n} 2\pi p(x_i) h(x_i) \Delta x$$

$$= \int_0^1 2\pi p(x) h(x)\, dx$$

$$= \int_0^1 2\pi x \left(e^{x^2} - 1\right) dx$$

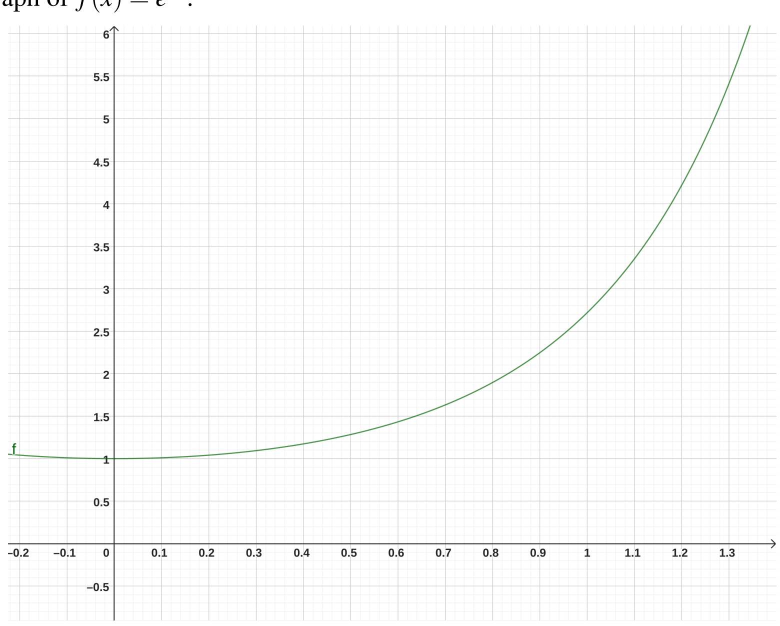
$$= 2\pi \left[\frac{e^{x^2}}{2} - \frac{x^2}{2} \right]_0^1$$

$$= \pi \left[e^{x^2} - x^2 \right]_0^1$$

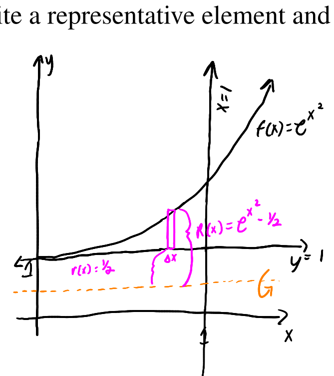
$$= \pi \left[\left[e^{1^2} - 1 \right] - \left[e^0 - 0 \right] \right]$$

$$= \pi \left[e - 2 \right]$$

Question 4. *(5 marks)* Given the graph of $f(x) = e^{x^2}$.



Consider the region in the first quadrant $\mathscr{R}$ bounded by $y = e^{x^2}$, $x = 1$ and $y = 1$. Using discs or washers, set up, **but do not evaluate**, the integral required to find the volume of the solid obtained by revolving the region $\mathscr{R}$ about the line $y = \frac{1}{2}$. Sketch the region, draw a representative rectangle, write a representative element and label the sketch completely.



Volume of a single washer: $\Delta V = \pi \left[(R(x))^2 - (r(x))^2 \right] \Delta x$

$$\text{Volume} = \lim_{n \to \infty} \sum_{i=1}^{n} \pi \left[(R(x_i))^2 - (r(x_i))^2 \right] \Delta x$$

$$= \int_0^1 \pi \left[(R(x_i))^2 - (r(x_i))^2 \right] dx$$

$$= \int_0^1 \pi \left[\left(e^{x^2} - \tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2 \right] dx$$