Dawson College: **Integral Calculus**: 201-SN3-RE-S2: Winter 2026: **Quiz 6** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* A sequence begins

$$6,\ 7,\ \frac{6}{2},\ \frac{7}{6},\ \frac{6}{24},\ \frac{7}{120},\ \frac{6}{720},\ldots$$

Find a formula for the general term a_n.

$$\{a_n\}_{n=0}^{\infty} = \left\{ \frac{(13-(-1)^n)/2}{n!} \right\}_{n=0}^{\infty}$$

Question 2. *(5 marks)* Use the Squeeze Theorem to find

$$\lim_{n\to\infty} \underbrace{\frac{\left(\cos\left(\frac{1}{n}\right)\right)^n}{n!}}_{a_n}.$$

$$a_n \geq 0 = b_n, \quad \lim_{n\to\infty} b_n = 0$$

$$a_n = \frac{\left(\cos\left(\frac{1}{n}\right)\right)^n}{n!} \leq \frac{1^n}{n!} = \frac{1}{n!} = \frac{1}{1\cdot 2\cdot 3\cdots n} \leq \frac{1}{n} = c_n$$

$$\lim_{n\to\infty} c_n = 0$$

$\therefore$ by the squeeze thm. $\lim\limits_{n\to\infty} a_n = 0$ since $b_n \leq a_n \leq c_n \ \forall n$ and $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} c_n = 0$

Question 3. *(5 marks)* Determine whether the sequence

$$a_n = \left(\frac{1}{n}\right)^{1/\ln(\ln n)}$$

converges or diverges. If it converges, find its limit.

$$\lim_{n\to\infty} a_n$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right)^{1/\ln(\ln n)}$$

$$= \lim_{x\to\infty} \left(\frac{1}{x}\right)^{\frac{1}{\ln(\ln x)}}$$

$$= 0$$

$$y = \lim_{x\to\infty} \left(\frac{1}{x}\right)^{1/\ln(\ln x)}$$

$$\ln y = \ln \lim_{x\to\infty} \left(\frac{1}{x}\right)^{\frac{1}{\ln(\ln x)}}$$

$$\ln y = \lim_{x\to\infty} \ln \left(\frac{1}{x}\right)^{\frac{1}{\ln(\ln x)}}$$

$$\ln y = \lim_{x\to\infty} \frac{1}{\ln(\ln x)} \ln\left(\frac{1}{x}\right)$$

$$\ln y = \lim_{x\to\infty} \frac{-\ln x}{\ln(\ln x)} \quad \text{I.F. } \frac{-\infty}{\infty}$$

$$\ln y \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{-\frac{1}{x}}{\frac{1}{\ln x}\frac{1}{x}}$$

$$\ln y = \lim_{x\to\infty} -\ln x = -\infty$$

since $\ln y \to -\infty$ as $x \to \infty$ we have that $y \to 0$ as $x \to \infty$

Question 4. *(5 marks each)* Determine whether the series

a.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{2}}$$

Lets apply the n^{th} term div. test

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\sqrt[n]{2}} = \lim_{n\to\infty} \frac{1}{2^{1/n}} = 1 \neq 0$$

$\therefore$ the series div.

b.

$$\sum_{n=2}^{\infty} 2026\left(\frac{1}{2}\right)^{n-1}$$

converges or diverges. If it converges, find its sum.

$$\sum_{n=2}^{\infty} 2026\left(\frac{1}{2}\right)^{n-1-1+1}$$ conv. since geometric series where $|r| = \frac{1}{2} < 1$

$$= \sum_{n=2}^{\infty} 2026\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-2}$$

$$= \frac{a}{1-r} = \frac{2026(1/2)}{1-1/2} = 2026$$

Question 5. *(5 marks)* The nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is

$$s_n = \frac{n+2}{n+3}.$$

1. Find a_1. $S_1 = a_1 \therefore a_1 = \frac{1+2}{1+3} = \frac{3}{4}$
2. Find a formula for a_n for $n \geq 2$.
3. Identify the type of series.
4. Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.
5. If it converges, find its sum. $S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n+2}{n+3} = 1$ $\therefore$ series converges to 1.

① $S_{n-1} = a_1 + a_2 + \dots + a_{n-1}$

② $S_n = a_1 + a_2 + \dots + a_{n-1} + a_n$

② - ①

$$S_n - S_{n-1} = a_n$$

$$a_n = \frac{n+2}{n+3} - \frac{(n-1)+2}{(n-1)+3} = \frac{n+2}{n+3} - \frac{n+1}{n+2}$$

The series is a telescoping series since

$$S_n = a_1 + a_2 + a_3 + \dots + a_{n-2} + a_{n-1} + a_n$$

$$= \cancel{\frac{3}{4}} + \left[\cancel{\frac{4}{5}} - \cancel{\frac{3}{4}}\right] + \left[\cancel{\frac{5}{6}} - \cancel{\frac{4}{5}}\right] + \dots + \left[\cancel{\frac{n}{n+1}} - \cancel{\frac{n-1}{n+1}}\right] + \left[\cancel{\frac{n+1}{n+2}} - \cancel{\frac{n}{n+1}}\right] + \left[\frac{n+2}{n+3} - \cancel{\frac{n+1}{n+2}}\right]$$