

Question 1. (3 marks each) Determine whether the following statements are true or false. If a statement is false provide a counterexample. If it is true provide a proof.

a. Consider a linear system with augmented matrix A . If the system has a **unique** solution, then in the **reduced row echelon form** of A every column contains a leading 1.

b. If the **row echelon form** of an augmented matrix contains a row of zeros, then the corresponding system has infinitely many solutions.

Question 2. (3 marks) Find (if possible) conditions on a and b such that the system has no solution, exactly one solution, and infinitely many solutions. Justify.

$$\begin{cases} ax + 2y = 1 \\ bx + 4y = 2 \end{cases}$$

Question 3. (2 marks) Consider the following augmented matrix of a **consistent** linear system.

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ -1 & 1 & 0 \end{bmatrix}$$

Find a row which can be removed from the augmented matrix to make a new system with two equations which has the **same solution set**. Justify.

Question 4. (3 marks each) Each matrix below is in **REF** or **RREF** and represents an augmented matrix for a linear system. For each one, determine whether the system has **no solution**, **exactly one solution**, or **infinitely many solutions**. When solutions exist, write the **solution set**.

a. (involving a parameter k)

$$\begin{bmatrix} 1 & 0 & 2 & -1 & 3 \\ 0 & 1 & -4 & 5 & -2 \\ 0 & 0 & 0 & 0 & k \end{bmatrix}$$

(State for which values of k the system is consistent, then give the solution set when it is consistent.)

b.

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Question 5. (3 marks) Find the solution set of the following equation:

$$x_1 - 2x_2 + 3x_3 + \mu x_4 = 6.$$

Also find the value of μ such that $(x_1, x_2, x_3, x_4) = (1, 0, 1, 2)$ is a particular solution.