Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statements are true or false. If a statement is false provide a counterexample. If it is true provide a proof.

a. Consider a linear system with augmented matrix A. If the system has a **unique** solution, then in the **reduced row echelon form** of A every column contains a leading 1.

False, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ has a unique solution but no leading 1 in constant column.

b. If the **row echelon form** of an augmented matrix contains a row of zeros, then the corresponding system has infinitely many solutions.

False, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ has a row of zeros but is inconsistent since $0x_1 = 1$ cannot be satisfied.

Question 2. *(3 marks)* Find (if possible) conditions on a and b such that the system has no solution, exactly one solution, and infinitely many solutions. Justify.

Two lines on the plane

$$\begin{cases} ax + 2y = 1 \\ bx + 4y = 2 \end{cases}$$

$y = -\frac{a}{2}x + \frac{1}{2}$

$y = -\frac{b}{4}x + \frac{1}{2}$

The y-intercepts of the lines are always the same. $\therefore$ the system is always consistent because they always have the intercept in common.

If $-\frac{a}{2} \neq -\frac{b}{4}$ or $2a \neq b$ then the two lines have different slopes and they intersect at the y-intercept only. $\therefore$ a unique solution.

If $-\frac{a}{2} = -\frac{b}{4}$ or $2a = b$ then the two line are identical and all their points of their graph are in common. $\therefore$ infinitely many solutions.

Question 3. *(2 marks)* Consider the following augmented matrix of a **consistent** linear system.

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ -1 & 1 & 0 \end{bmatrix}$$

Find a row which can be removed from the augmented matrix to make a new system with two equations which has the **same solution set**. Justify.

The equations of row 1 and 2 $\begin{matrix} x-2y=3 \\ 2x-4y=6 \end{matrix}$ are identical lines.

$\therefore$ removing row 1 or row 2 will not change the solution set.

$\therefore \begin{bmatrix} 1 & -2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$

Question 4. *(3 marks each)* Each matrix below is in **REF** or **RREF** and represents an augmented matrix for a linear system. For each one, determine whether the system has **no solution**, **exactly one solution**, or **infinitely many solutions**. When solutions exist, write the **solution set**.

a. **(involving a parameter k)**

$$\begin{bmatrix} 1 & 0 & 2 & -1 & 3 \\ 0 & 1 & -4 & 5 & -2 \\ 0 & 0 & 0 & 0 & k \end{bmatrix}$$

If $k \neq 0$ then the system is inconsistent because of leading entry in constant column.

(State for which values of k the system is consistent, then give the solution set when it is consistent.)

If $k=0$, let $x_3 = s$, $x_4 = t$, $s,t \in \mathbb{R}$ $\Rightarrow$ $x_1 = 3-2s+t$, $x_2 = -2+4s-5t$

$\therefore (x_1, x_2, x_3, x_4) = (3-2s+t, -2+4s-5t, s, t)$ $s,t \in \mathbb{R}$

b.

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

1 $x_1 + 2x_2 - x_3 = 3$

2 $x_2 + 4x_3 = -2$

3 $x_3 = 5$

sub 3 into 2: $x_2 + 4(5) = -2$

$x_2 = -22$ 4

sub 3, 4 into 1

$x_1 + 2(-22) - (5) = 3$

$x_1 = 52$

$\therefore (x_1, x_2, x_3) = (52, -22, 5)$

Question 5. *(3 marks)* Find the solution set of the following equation:

$$x_1 - 2x_2 + 3x_3 + \mu x_4 = 6.$$

Also find the value of μ such that $(x_1, x_2, x_3, x_4) = (1, 0, 1, 2)$ is a particular solution.

Let $x_2 = r$, $x_3 = s$, $x_4 = t$

$x_1 - 2r + 3s + \mu t = 6$

$x_1 = 6 + 2r - 3s - \mu t$

$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 6+2r-3s-\mu t \\ r \\ s \\ t \end{pmatrix}$ $r,s,t \in \mathbb{R}$

$(1,0,1,2)$ must satisfy the linear equation.

$1 - 2(0) + 3(1) + \mu(2) = 6$

$\mu(2) = 2$

$\mu = 1$