

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531***. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (3 marks each) Determine whether the following statements are true or false. If a statement is false provide a counterexample. If it is true provide a proof.

a. If AB and BA are both defined, then AB and BA are square matrices.

b. If A and B are square matrices of the same size then $\text{tr}(AB) = \text{tr}(A)\text{tr}(B)$.

Question 2. (5 marks) Given $abc \neq 0$ find the reduced row echelon form of

$$\begin{bmatrix} p & 0 & a \\ b & 0 & 0 \\ q & c & r \end{bmatrix}$$

Question 3. (6 marks) determine for what values of k , if any, the following linear system has exactly one solution, no solutions, and infinitely many solutions.

$$\begin{bmatrix} 1 & 1 & k & 1 \\ 1 & k+1 & 2k+1 & 3 \\ -k & -4k & -3 & -6 \end{bmatrix}$$

Question 4. (5 marks) If $U = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$, and $AU = 0$, show that $A = 0$ where $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Bonus Question. (3 marks) If A and B are matrices such that the operations are defined, show that $(AB)^T = B^T A^T$.