Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 2** name: ____________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statements are true or false. If a statement is false provide a counterexample. If it is true provide a proof.

a. If AB and BA are both defined, then AB and BA are square matrices.

True, Let A be a $m\times n$ matrix

B " " $p\times q$ "

since $A_{m\times n}B_{p\times q}$ is defined $n=p$

" $B_{p\times q}A_{m\times n}$ " " $q=m$

Then the result of $A_{m\times n}B_{p\times q}$ is an $m\times q$, but $q=m$ $\therefore$ $m\times m$

And $B_{p\times q}A_{m\times n}$ results in a $p\times n$

but $p=n$ $\therefore$ $n\times n$

b. If A and B are square matrices of the same size then $\text{tr}(AB)=\text{tr}(A)\text{tr}(B)$.

False, Let $A=\begin{bmatrix}1&0\\0&1\end{bmatrix}=B$

$$LHS = tr(AB) = tr\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\right) = tr\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)=2$$

$$RHS = tr(A)tr(B) = 2(2) = 4 \neq LHS$$

Question 2. *(5 marks)* Given $abc\neq 0$ find the reduced row echelon form of

since $abc\neq 0 \Rightarrow a\neq 0,\ b\neq 0,\ c\neq 0$

$$\begin{bmatrix}p&0&a\\b&0&0\\q&c&r\end{bmatrix} \sim_{R_1\leftrightarrow R_2} \begin{bmatrix}b&0&0\\p&0&a\\q&c&r\end{bmatrix} \sim_{\frac{1}{b}R_1\to R_1} \begin{bmatrix}1&0&0\\p&0&a\\q&c&r\end{bmatrix}$$

$$\sim_{\substack{-pR_1+R_2\to R_2\\ -qR_1+R_3\to R_3}} \begin{bmatrix}1&0&0\\0&0&a\\0&c&r\end{bmatrix} \sim_{R_2\leftrightarrow R_3} \begin{bmatrix}1&0&0\\0&c&r\\0&0&a\end{bmatrix}$$

$$\sim_{\frac{1}{a}R_3\to R_3} \begin{bmatrix}1&0&0\\0&c&r\\0&0&1\end{bmatrix} \sim_{-rR_3+R_2\to R_2} \begin{bmatrix}1&0&0\\0&c&0\\0&0&1\end{bmatrix} \sim_{\frac{1}{c}R_2\to R_2} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Question 3. *(6 marks)* etermine for what values of k, if any, the following linear system has exactly one solution, no solutions, and infinitely many solutions.

$$\begin{bmatrix} 1 & 1 & k & 1 \\ 1 & k+1 & 2k+1 & 3 \\ -k & -4k & -3 & -6 \end{bmatrix} \sim \begin{matrix} -R_1+R_2 \to R_2 \\ KR_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & K & 1 \\ 0 & K & K+1 & 2 \\ 0 & -3K & K^2-3 & K-6 \end{bmatrix} \sim 3R_2+R_3 \to R_3 \begin{bmatrix} 1 & 1 & K & 1 \\ 0 & K & K+1 & 2 \\ 0 & 0 & K^2+3K & K \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & K & 1 \\ 0 & K & K+1 & 2 \\ 0 & 0 & K(K+3) & K \end{bmatrix}$$

If $K=0$ then $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ #leading entry < #var and no leading entry in constant column. ∴ ∞ many solutions.

If $K=-3$ then $\begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & -3 & -2 & 2 \\ 0 & 0 & 0 & -6 \end{bmatrix}$ and the system has no solutions since there is a leading entry in the constant column.

If $K \neq 0,-3$ then a unique solution since #leading entry in variable column = #var.

Question 4. *(5 marks)* If $U = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$, and $AU = 0$, show that $A = 0$ where $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad AU = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} a & 2a-b \\ c & 2c-d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$a=0 \qquad 2a-b=0 \Rightarrow 2(0)-b=0 \Rightarrow b=0$

$c=0 \qquad 2c-d=0 \Rightarrow 2(0)-d=0 \Rightarrow d=0$

$\therefore A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Bonus Question. *(3 marks)* If A and B are matrices such that the operations are defined, show that $(AB)^T = B^T A^T$.