Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statements are true or false. If a statement is false provide a counterexample. If it is true provide a proof.

a. If $A^4 = 2I$, then the matrix A is invertible.

True, from the premise $\frac{1}{2}A^4 = I$

$\frac{1}{2}A^3A = I$ and $A\left(\frac{1}{2}A^3\right) = I$

$\therefore A$ is invertible and $A^{-1} = \frac{1}{2}A^3$

b. If $AB = B$ and B is a nonzero matrix, then the matrix A is invertible.

False, let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \neq 0$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = B$ but A is singular since $ad-bc$

$= 1(0) - 0(0)$

$= 0$

Question 2. *(4 marks)* Solve for the matrix X if $AX(D+BX)^{-1} = C$. Assume that all matrices are $n \times n$ and invertible as needed.

$$
\begin{aligned}
AX(D+BX)^{-1}(D+BX) &= C(D+BX) \\
AXI &= CD + CBX \\
AX - CBX &= CD \\
(A-CB)X &= CD \\
(A-CB)^{-1}(A-CB)X &= (A-CB)^{-1}CD \\
IX &= (A-CB)^{-1}CD \\
X &= (A-CB)^{-1}CD
\end{aligned}
$$

Question 3. *(5 marks)* Let $B^{-1}A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ and $BX^{-1}A^{-1} = I_3$. Find the matrix X.

$$B^{-1}BX^{-1}A^{-1}A = B^{-1}IA$$
$$IX^{-1}I = B^{-1}A$$
$$X^{-1} = B^{-1}A$$
$$(X^{-1})^{-1} = (B^{-1}A)^{-1}$$
$$X = (B^{-1}A)^{-1}$$

so lets find the inverse of the given matrix.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \begin{matrix} -R_1+R_2 \to R_2 \\ -2R_1+R_3 \to R_3 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\sim \; -3R_2+R_3 \to R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -3 & 1 \end{array}\right]$$

$$\sim \begin{matrix} R_3+R_1 \to R_1 \\ -R_2 \to R_2 \\ -R_3 \to R_3 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 2 & -3 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 3 & -1 \end{array}\right] \sim \; -2R_2+R_1 \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 3 & -1 \end{array}\right]$$

$$\therefore \; X = (B^{-1}A)^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -1 & 0 \\ -1 & 3 & -1 \end{bmatrix}$$

Question 4. *(5 marks)* Express the above $B^{-1}A$ as a product of elementary matrices, if possible.

Since the reduced row echelon form is I, it is possible

$$E_7 \cdots E_2E_1B^{-1}A = I$$
$$(E_7 \cdots E_2E_1)^{-1}E_7 \cdots E_2E_1B^{-1}A = (E_7 \cdots E_2E_1)^{-1}I$$
$$B^{-1}A = E_1^{-1}E_2^{-1} \cdots E_7^{-1}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$