Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 4** name: ______________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statements are true or false. If a statement is false provide a counterexample. If it is true provide a proof.

a. If A is row equivalent to I_n, and E is an elementary matrix, then the system $EA^2A^T\mathbf{x} = \mathbf{b}$ is consistent for any $n \times 1$ column matrix $\mathbf{b}$.

b. If $\mathbf{x}_1$ is any solution to the linear system $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_0$ is any solution to the associated homogeneous system $A\mathbf{x} = \mathbf{0}$ then $\mathbf{x}_k = \mathbf{x}_1 + k\mathbf{x}_0$ for any scalar k is a solution to the linear system $A\mathbf{x} = \mathbf{b}$.

Question 2. *(3 marks)* Prove: If $A^TA = A$, then A is symmetric and $A = A^2$.

Question 3. *(3 marks)* An $n \times n$ matrix P is called an *idempotent matrix* if $P^2 = P$. Show using a proof by contradiction that I is the only invertible idempotent matrix.

Question 4. *(3 marks)* Find all 2×2 skew symmetric matrices A such that $A^2 = -I$.