Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 5** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark per blank)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If A is a product of elementary matrices, then $\det(A)$ ______________ equal zero.

b. If A and B are square matrices of the same size then $\det(A+B)$ ______________ be equal to $\det(A)+\det(B)$.

c. If A is an $n\times r$ then $\det(AA^T)$ ______________ be equal $(\det(A))^2$.

Question 2. *(1 mark per blank)* Given A an $n\times n$ matrix and k a non-zero scalar.

a. If A is an elementary matrix obtained by interchanging two rows then $\det(A) =$ ______________.

b. If A is the reduced row echelon form of an invertible matrix then $\det(A) =$ ______________.

c. If A is a singular matrix then $\det(A) =$ ______________.

d. If A is an elementary matrix obtained by adding k times one row to an other then $\det(A) =$ ______________.

e. If A is an elementary matrix obtained by multiplying one row by k then $\det(A) =$ ______________.

f. If A is the identity matrix multiplied by k then $\det(A) =$ ______________.

Question 3. Let A and B be two 3×3 matrices such that $\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$, and $\det(B) = -4$. Find the following:

a. *(4 marks)* $\begin{vmatrix} a+b & 3d+3e & g+h \\ 2a-3b & 6d-9e & 2g-3h \\ 2c & 6f & 2i \end{vmatrix}$.

b. *(3 marks)* $\det\left(5B^{-1}A + \operatorname{adj}(A^{-1}B)\right)$.

[1] From or modified from a John Abbott final examination

Question 4. *(4 marks)* Evaluate the following determinant. (Hint: first use elementary operations)

$$\begin{vmatrix} 3 & 2 & 5 & 6 \\ 8 & 4 & 8 & 9 \\ -7 & -6 & -11 & -19 \\ -3 & -2 & 3 & -4 \end{vmatrix}$$

Question 5. *(4 marks)* Let A be a 3×3 invertible matrix such that $A^2 = \operatorname{adj}(-2A^{-1})$. Find $\det(A)$.

Question 6. *(4 marks)* Find all values of t, if any, for which A is singular where

$$A = \begin{bmatrix} 2-t & -1 & 0 \\ 1 & 3+t & -2-t \\ -1 & 0 & 2+t \end{bmatrix}$$