Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark per blank)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If A is a product of elementary matrices, then $\det(A)$ cannot equal zero.

b. If A and B are square matrices of the same size then $\det(A+B)$ might be equal to $\det(A)+\det(B)$.

c. If A is an $n \times r$ then $\det(AA^T)$ might be equal $(\det(A))^2$.

Question 2. *(1 mark per blank)* Given A an $n \times n$ matrix and k a non-zero scalar.

a. If A is an elementary matrix obtained by interchanging two rows then $\det(A) =$ -1.

b. If A is the reduced row echelon form of an invertible matrix then $\det(A) =$ 1.

c. If A is a singular matrix then $\det(A) =$ 0.

d. If A is an elementary matrix obtained by adding k times one row to an other then $\det(A) =$ 1.

e. If A is an elementary matrix obtained by multiplying one row by k then $\det(A) =$ k.

f. If A is the identity matrix multiplied by k then $\det(A) =$ k^n.

Question 3. Let A and B be two 3×3 matrices such that $\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$, and $\det(B) = -4$. Find the following:

a. *(4 marks)* $\begin{vmatrix} a+b & 3d+3e & g+h \\ 2a-3b & 6d-9e & 2g-3h \\ 2c & 6f & 2i \end{vmatrix}$.

$$= \underset{\frac{1}{3}C_2 \to C_2}{\tfrac{1}{2}R_3 \to R_3} \; 6\begin{vmatrix} a+b & d+e & g+h \\ 2a-3b & 2d-3e & 2g-3h \\ c & f & i \end{vmatrix}$$

$$= -2R_1+R_2 \to R_2 \; 6\begin{vmatrix} a+b & d+e & g+h \\ -5b & -5e & -5h \\ c & f & i \end{vmatrix}$$

$$= -\tfrac{1}{5}R_2 \to R_2 \; -30\begin{vmatrix} a+b & d+e & g+h \\ b & e & h \\ c & f & i \end{vmatrix}$$

$$= -R_2+R_1 \to R_1 \; -30\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$$

transpose does not change the det.

$$= -30\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -60$$

b. *(3 marks)* $\det\left(5B^{-1}A + \operatorname{adj}(A^{-1}B)\right)$.

$(A^{-1}B)^{-1} = \dfrac{1}{\det(A^{-1}B)} \operatorname{adj}(A^{-1}B)$

$\operatorname{adj}(A^{-1}B) = \det(A^{-1}B)\, B^{-1}(A^{-1})^{-1}$

$\operatorname{adj}(A^{-1}B) = \det(A^{-1})\det(B)\, B^{-1}A$

$\operatorname{adj}(A^{-1}B) = \dfrac{1}{\det A} \cdot -4\, B^{-1}A$

$\operatorname{adj}(A^{-1}B) = -2B^{-1}A$

$\det(5B^{-1}A + \operatorname{adj}(A^{-1}B))$

$= \det(5B^{-1}A - 2B^{-1}A)$

$= \det(3B^{-1}A)$

$= 3^3 \det(B^{-1}A)$

$= 3^3 \det(B^{-1}) \det A$

$= 3^3 \dfrac{1}{\det B} \det A$

$= 3^3 \left(\dfrac{1}{4}\right)^{-2}$

$= -\dfrac{3^3}{2}$

[1] From or modified from a John Abbott final examination

Question 4. *(4 marks)* Evaluate the following determinant. (Hint: first use elementary operations)

$$\begin{vmatrix} 3 & 2 & 5 & 6 \\ 8 & 4 & 8 & 9 \\ -7 & -6 & -11 & -19 \\ -3 & -2 & 3 & -4 \end{vmatrix} \underset{\substack{-2R_1+R_2\to R_2 \\ 3R_1+R_3\to R_3 \\ R_1+R_4\to R_4}}{=} \begin{vmatrix} 3 & 2 & 5 & 6 \\ 2 & 0 & -2 & -3 \\ 2 & 0 & 4 & -1 \\ 0 & 0 & 8 & 2 \end{vmatrix}$$

$$= 2(-1)^{1+2} \begin{vmatrix} 2 & -2 & -3 \\ 2 & 4 & -1 \\ 0 & 8 & 2 \end{vmatrix}$$

$$= -2 \underset{-R_1+R_2\to R_2}{} \begin{vmatrix} 2 & -2 & -3 \\ 0 & 6 & 2 \\ 0 & 8 & 6 \end{vmatrix}$$

$$= -2(2)(-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 8 & 6 \end{vmatrix}$$

$$= -4\left[6(6) - 2(8)\right]$$

$$= -80$$

Question 5. *(4 marks)* Let A be a 3×3 invertible matrix such that $A^2 = \operatorname{adj}(-2A^{-1})$. Find $\det(A)$.

$$\det(A^2) = \det(\operatorname{adj}(-2A^{-1}))$$

$$(\det(A))^2 = (\det(-2A^{-1}))^{3-1}$$

$$\det(A) = \pm \det(-2A^{-1})$$

$$\det(A) = \pm(-2)^3 \det(A^{-1})$$

$$\det(A) = \mp 8 \frac{1}{\det A}$$

$$(\det(A))^2 = 8$$

$$\det(A) = \pm\sqrt{8}$$

Question 6. *(4 marks)* Find all values of t, if any, for which A is singular where

$$A = \begin{bmatrix} 2-t & -1 & 0 \\ 1 & 3+t & -2-t \\ -1 & 0 & 2+t \end{bmatrix}$$

$$0 = |A|$$

$$0 = 0C_{13} + (-2-t)C_{23} + (2+t)C_{33}$$

$$0 = -(2+t)(-1)^{2+3}\begin{vmatrix} 2-t & -1 \\ -1 & 0 \end{vmatrix} + (2+t)(-1)^{3+3}\begin{vmatrix} 2-t & -1 \\ 1 & 3+t \end{vmatrix}$$

$$0 = (2+t)\left[-1 + (2-t)(3+t) + 1\right]$$

$$0 = (2+t)(2-t)(3+t)$$

$t=-2 \quad t=2 \quad t=-3$

$\therefore$ singular for the above values