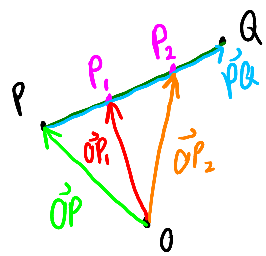
Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 6** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Using vectors find the two points trisecting[1] the segment between $P(2,3,5)$ and $Q(8,-6,2)$.



$$\vec{PQ} = \vec{OQ} - \vec{OP} = (8,-6,2)-(2,3,5) = (6,-9,-3)$$

$$\vec{OP_1} = \vec{OP} + \vec{PP_1} = \vec{OP} + \tfrac{1}{3}\vec{PQ} = (2,3,5) + \tfrac{1}{3}(6,-9,-3) = (4,0,4)$$

$\therefore$ $P_1(4,0,4)$

$$\vec{OP_2} = \vec{OP} + \vec{PP_2} = \vec{OP} + \tfrac{2}{3}\vec{PQ} = (2,3,5) + \tfrac{2}{3}(6,-9,-3) = (6,-3,3)$$

$\therefore$ $P_2(6,-3,3)$

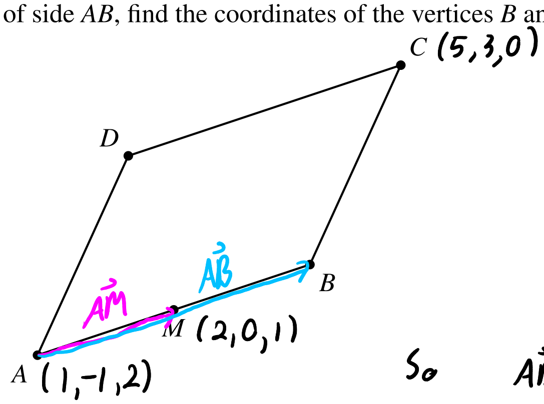
Question 2. *(5 marks)* A parallelogram has vertices A, B, C, and D in order. Given

$A(1,-1,2)$, $C(5,3,0)$,

and the midpoint

$M(2,0,1)$

of side AB, find the coordinates of the vertices B and D.



$$\vec{AB} = 2\vec{AM} = 2(\vec{OM} - \vec{OA}) = 2((2,0,1)-(1,-1,2)) = 2(1,1,-1) = (2,2,-2)$$

So $\vec{AB} = (2,2,-2)$

$$\vec{OB} - \vec{OA} = (2,2,-2)$$

$$\vec{OB} = \vec{OA} + (2,2,-2) = (1,-1,2) + (2,2,-2) = (3,1,0)$$

$\therefore$ $B(3,1,0)$

$$\vec{AB} = \vec{DC}$$

$$\vec{AB} = \vec{OC} - \vec{OD}$$

$$\vec{OD} = \vec{OC} - \vec{OA} = (5,3,0) - (2,2,-2) = (3,1,2)$$

$\therefore$ $D(3,1,2)$

[1] dividing something into three equal parts.

Question 3. *(5 marks)* If $\mathbf{u} = (0,1,1)$ and $\mathbf{v} = (p,4,p)$, find the parameter p such that the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\pi/3$.

$$\underline{u} \cdot \underline{v} = \|\underline{u}\|\|\underline{v}\| \cos\theta$$

$$(0,1,1)\cdot(p,4,p) = \sqrt{0^2+1^2+1^2}\sqrt{p^2+4^2+p^2}\cos\frac{\pi}{3}$$

$$4+p = \sqrt{2}\sqrt{2p^2+16}\ \frac{1}{2}$$

$$4+p = \sqrt{2}\sqrt{2}\sqrt{p^2+8}\ \frac{1}{2}$$

$$4+p = \sqrt{p^2+8}$$

$$(4+p)^2 = p^2+8$$

$$p^2+8p+16 = p^2+8$$

$$8p = -8$$

$$p = -1$$

Question 4. Given that $\|\mathbf{x}\| = 1$, $\|\mathbf{y}\| = 3$, and $\mathbf{x}\cdot\mathbf{y} = 2$, find

a. *(2 marks)* $4\mathbf{x}\cdot(\mathbf{y}-2\mathbf{x})$ $= 4\underline{x}\cdot\underline{y} - (4\underline{x})\cdot(2\underline{x}) = 4(\underline{x}\cdot\underline{y}) - 8(\underline{x}\cdot\underline{x}) = 4(2) - 8\|\underline{x}\|^2$

$= 8 - 8(1)^2$

$= 0$

b. *(3 marks)* $\|\mathbf{x}+\mathbf{y}\|$

$$\|\underline{x}+\underline{y}\|^2 = (\underline{x}+\underline{y})\cdot(\underline{x}+\underline{y})$$

$$= \underline{x}\cdot\underline{x} + \underline{x}\cdot\underline{y} + \underline{y}\cdot\underline{x} + \underline{y}\cdot\underline{y}$$

$$= \|\underline{x}\|^2 + \underline{x}\cdot\underline{y} + \underline{x}\cdot\underline{y} + \|\underline{y}\|^2$$

$$= \|\underline{x}\|^2 + 2\underline{x}\cdot\underline{y} + \|\underline{y}\|^2$$

$$= 1^2 + 2(2) + 3^2$$

$$= 14$$

$$\therefore \|\underline{x}+\underline{y}\| = \sqrt{14}$$