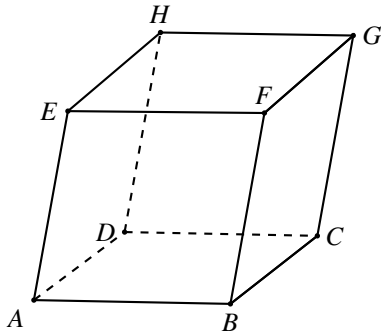


Question 1.

a. (2 marks) State a formula for the area of the triangle in 2-space determined by the vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$.

b. (2 marks) State a formula for the area of the parallelogram in 3-space determined by the vertices A , B , C , and D .

c. (3 marks) State a formula for the volume of the parallelepiped described below.



Bonus. (5 marks) Precisely explain how to compute the 3-dimensional volume of a parallelepiped determined by three vectors in \mathbb{R}^4

Question 2. (5 marks) Let $\mathcal{L}_1 : (x, y, z) = (1, -2, 3) + t(2, 1, -1)$ where $t \in \mathbb{R}$ and let \mathcal{L}_2 be the line of intersection of the planes $x + 2z = 0$ and $y + z = 1$. Find the shortest distance between \mathcal{L}_1 and \mathcal{L}_2 .

Question 3. (5 marks) Find both the parametric and general equation of the plane containing the point $A(2, -1, 4)$, parallel to the line $\mathcal{L} : (x, y, z) = (1, 0, 2) + t(2, 1, 0)$ where $t \in \mathbb{R}$ and perpendicular to the plane $\mathcal{P} : x - 2y + 2z = 5$.

Question 4. (5 marks) Let $\mathcal{L} : (x, y, z) = (3, -1, 2) + t(1, 2, -1)$ where $t \in \mathbb{R}$ and let \mathcal{P} be the plane passing through the points $A(1, 0, 1)$, $B(2, 1, 3)$, $C(0, 2, 2)$. Determine whether \mathcal{L} intersects \mathcal{P} . If it does, find the point of intersection. If it does not, determine whether \mathcal{L} is parallel to \mathcal{P} or contained in \mathcal{P} .

Question 5. (5 marks) Let $A(4, -1, 2)$ and let \mathcal{P} be the plane containing the line $\mathcal{L}_1 (x, y, z) = (1, 2, 0) + s(1, -1, 2)$ where $s \in \mathbb{R}$ and parallel to the line $\mathcal{L}_2 (x, y, z) = (0, 1, 3) + t(2, 1, -1)$ where $t \in \mathbb{R}$. Find the point on \mathcal{P} that is closest to A .