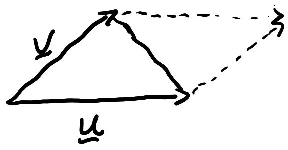
Dawson College: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 7** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

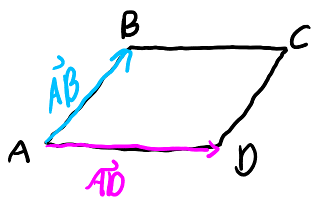
Question 1.

a. *(2 marks)* State a formula for the area of the triangle in 2-space determined by the vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$.



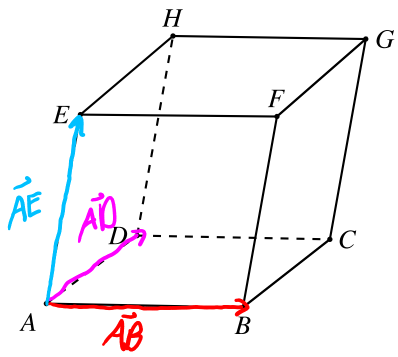
$$\text{Area} = \frac{\left|\det\left(\begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}\right)\right|}{2}$$

b. *(2 marks)* State a formula for the area of the parallelogram in 3-space determined by the vertices A, B, C, and D.



$$\text{Area} = \|\vec{AB} \times \vec{AD}\|$$

c. *(3 marks)* State a formula for the volume of the parallelepiped described below.



$$\text{Volume} = |\vec{AB} \cdot (\vec{AD} \times \vec{AE})|$$

Bonus. *(5 marks)* Precisely explain how to compute the 3-dimensional volume of a parallelepiped determined by three vectors in $\mathbb{R}^4$

Question 2. *(5 marks)* Let $\mathscr{L}_1 : (x, y, z) = \overbrace{(1, -2, 3)}^{P_1} + t\overbrace{(2, 1, -1)}^{d_1}$ where $t \in \mathbb{R}$ and let $\mathscr{L}_2$ be the line of intersection of the planes $x + 2z = 0$ and $y + z = 1$. Find the shortest distance between $\mathscr{L}_1$ and $\mathscr{L}_2$.

Lets find the intersection between the two planes

$x + 2z = 0$
$y + z = 1$

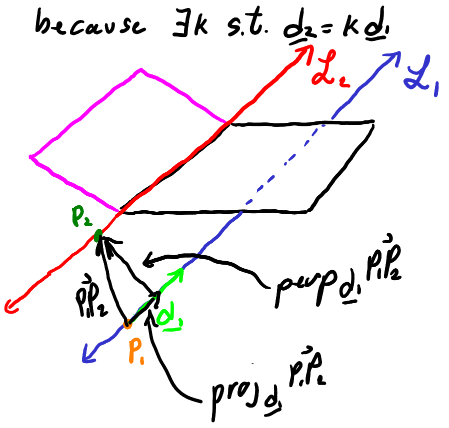
$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Let $z = t$ $t \in \mathbb{R}$

$x + 2t = 0 \Rightarrow x = -2t$
$y + t = 1 \Rightarrow y = 1 - t$

$\therefore (x, y, z) = (-2t, 1-t, t)$ $t \in \mathbb{R}$
$= \underbrace{(0, 1, 0)}_{P_2} + t\underbrace{(-2, -1, 1)}_{d_2}$

note: $\mathscr{L}_1 // \mathscr{L}_2$ since $d_1 // d_2$
because $\exists k$ s.t. $d_2 = k d_1$



distance $= \|\text{perp}_{d_1} \vec{P_1P_2}\|$

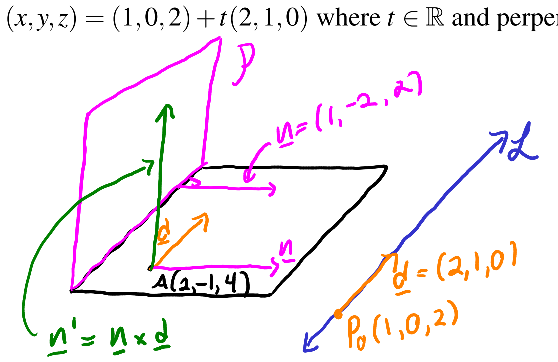
$\vec{P_1P_2} = \vec{OP_2} - \vec{OP_1} = (0, 1, 0) - (1, -2, 3)$
$= (-1, 3, -3)$

$$\text{proj}_{d_1} \vec{P_1P_2} = \frac{\vec{P_1P_2} \cdot d_1}{d_1 \cdot d_1} d_1$$
$$= \frac{(-1, 3, -3) \cdot (2, 1, -1)}{(2, 1, -1) \cdot (2, 1, -1)} (2, 1, -1)$$
$$= \frac{4}{6}(2, 1, -1)$$
$$= \frac{2}{3}(2, 1, -1)$$

$$\text{perp}_{d_1} \vec{P_1P_2} = \vec{P_1P_2} - \text{proj}_{d_1} \vec{P_1P_2}$$
$$= (-1, 3, -3) - \frac{2}{3}(2, 1, -1)$$
$$= \left(\frac{-7}{3}, \frac{7}{3}, \frac{-7}{3}\right)$$
$$= \frac{7}{3}(-1, 1, -1)$$

$$\text{distance} = \left\|\frac{7}{3}(-1, 1, -1)\right\|$$
$$= \frac{7}{3}\|(-1, 1, -1)\|$$
$$= \frac{7}{3}\sqrt{3}$$

Question 3. *(5 marks)* Find both the parametric and general equation of the plane containing the point $A(2,-1,4)$, parallel to the line $\mathscr{L}$: $(x,y,z) = (1,0,2) + t(2,1,0)$ where $t \in \mathbb{R}$ and perpendicular to the plane $\mathscr{P}$: $x - 2y + 2z = 5$.



$$\underline{x} = \underline{A} + s\underline{n} + t\underline{d} \quad \text{where } s,t \in \mathbb{R}$$
$$= (2,-1,4) + s(1,-2,2) + t(2,1,0)$$

$$\underline{n}' = \underline{n} \times \underline{d} = \left(\begin{vmatrix} -2 & 1 \\ 2 & 0 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} \right)$$
$$\begin{matrix} 1 & 2 \\ -2 & 1 \\ 2 & 0 \end{matrix} = (-2,4,5)$$

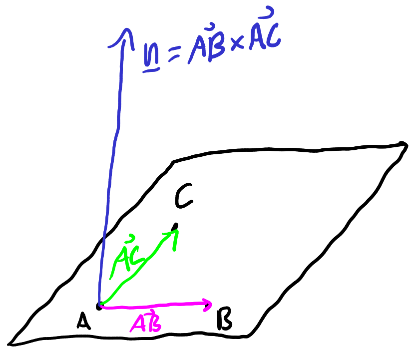
$$ax + by + cz = d$$
$$-2x + 4y + 5z = d$$

sub A

$$-2(2) + 4(-1) + 5(4) = d$$
$$d = 12$$

$$\therefore -2x + 4y + 5z = 12$$

Question 4. *(5 marks)* Let $\mathscr{L}$: $(x,y,z) = (3,-1,2) + t(1,2,-1)$ where $t \in \mathbb{R}$ and let $\mathscr{P}$ be the plane passing through the points $A(1,0,1)$, $B(2,1,3)$, $C(0,2,2)$. Determine whether $\mathscr{L}$ intersects $\mathscr{P}$. If it does, find the point of intersection. If it does not, determine whether $\mathscr{L}$ is parallel to $\mathscr{P}$ or contained in $\mathscr{P}$.



$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= (2,1,3) - (1,0,1) = (1,1,2)$$
$$\vec{AC} = \vec{OC} - \vec{OA}$$
$$= (0,2,2) - (1,0,1) = (-1,2,1)$$

$$\underline{n} = \vec{AB} \times \vec{AC} = \left(\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} \right) = (-3,-3,3)$$
$$\begin{matrix} 1 & -1 \\ 1 & 2 \\ 2 & 1 \end{matrix}$$

$$ax + by + cz = d$$
$$-3x - 3y + 3z = d$$

sub A to find d

$$-3(1) - 3(0) + 3(1) = d$$
$$0 = d$$
$$\therefore -3x - 3y + 3z = 0$$
$$x + y - z = 0$$

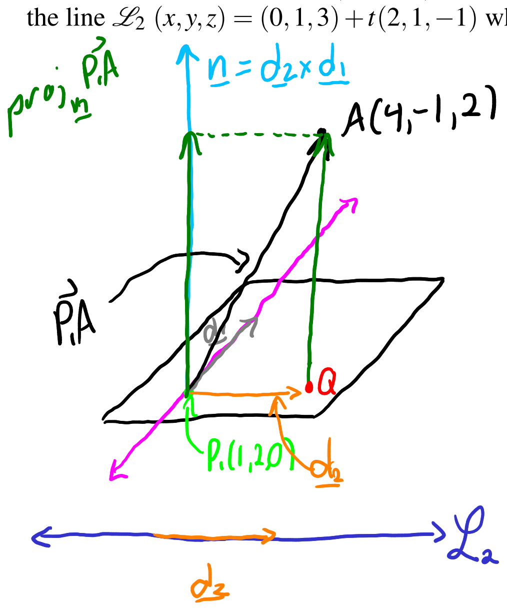
Sub $\mathscr{L}$ into $x + y - z = 0$

$$(3+t) + (-1+2t) - (2-t) = 0$$
$$4t = 0$$
$$t = 0$$

$$\therefore (x,y,z) = (3,-1,2) + 0(1,2,-1)$$
$$= (3,-1,2)$$

is the point of intersection.

Question 5. *(5 marks)* Let $A(4,-1,2)$ and let $\mathscr{P}$ be the plane containing the line $\mathscr{L}_1$ $(x,y,z) = (1,2,0) + s(1,-1,2)$ where $s \in \mathbb{R}$ and parallel to the line $\mathscr{L}_2$ $(x,y,z) = (0,1,3) + t(2,1,-1)$ where $t \in \mathbb{R}$. Find the point on $\mathscr{P}$ that is closest to A.



Lets find Q the closest point to A.

$$\vec{P_1A} = \vec{OA} - \vec{OP_1} = (4,-1,2) - (1,2,0) = (3,-3,2)$$

$$\underline{n} = \underline{d_2} \times \underline{d_1} = \left(\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} \right)$$
$$\begin{matrix} 2 & 1 \\ 1 & -1 \\ -1 & 2 \end{matrix} = (1,-5,-3)$$

$$\vec{QA} = proj_{\underline{n}} \vec{P_1A}$$
$$\vec{OA} - \vec{OQ} = proj_{\underline{n}} \vec{P_1A}$$
$$\vec{OQ} = \vec{OA} - proj_{\underline{n}} \vec{P_1A}$$
$$= (4,-1,2) - \frac{(3,-3,2)\cdot(1,-5,-3)}{(1,-5,-3)\cdot(1,-5,-3)}(1,-5,-3)$$
$$= (4,-1,2) - \frac{12}{35}(1,-5,3) = \left(\frac{128}{35}, \frac{25}{35}, \frac{106}{35}\right)$$