Dawson College: **Integral Calculus**: 201-SN3-RE-S2: Winter 2026: **Quiz 8** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(10 marks)* Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \underbrace{\frac{(x-2)^n}{n4^n}}_{a_n(x)}.$$

Lets find the radius of convergence by applying the Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}(x)}{a_n(x)}\right| = \lim_{n\to\infty}\left|\frac{\frac{(x-2)^{n+1}}{(n+1)4^{n+1}}}{\frac{(x-2)^n}{n4^n}}\right| = \lim_{n\to\infty}\left|\frac{(x-2)^{\cancel{n+1}}}{(n+1)4^{\cancel{n+1}}}\cdot\frac{n\cancel{4^n}}{\cancel{(x-2)^n}}\right|$$

$$= \lim_{n\to\infty}|x-2|\frac{n}{4(n+1)}$$

$$= |x-2|\lim_{n\to\infty}\frac{n}{4n+4}$$

$$= \frac{1}{4}|x-2| < 1$$

$$|x-2| < 4 = R$$

$\therefore$ radius of convergence $R=4$

$$-4 < x-2 < 4$$
$$-2 < x < 6$$

Test the endpoints:

$x=-2$: $\displaystyle\sum_{n=1}^{\infty}\frac{(-2-2)^n}{n4^n} = \sum_{n=1}^{\infty}\frac{(-4)^n}{n4^n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

Lets apply the Alternating Series Test for $b_n = \frac{1}{n}$

1) $\displaystyle\lim_{n\to\infty} b_n = 0$

2) $\frac{1}{n+1} < \frac{1}{n}$ $\therefore$ $b_{n+1} < b_n$

$\therefore$ convergence when $x=-2$

$x=6$: $\displaystyle\sum_{n=1}^{\infty}\frac{(6-2)^n}{n4^n} = \sum_{n=1}^{\infty}\frac{1}{n}$ is the harmonic series $\therefore$ diverges

$\therefore$ Interval of convergence is $[-2,6)$

Question 2. *(5 marks)* Find the Maclaurin series for $f(x) = e^{-2x}$ using the definition of a Maclaurin series.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\, x^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n x^n}{n!}$$

$$f^{(0)}(x) = e^{-2x}$$
$$f^{(1)}(x) = -2e^{-2x}$$
$$f^{(2)}(x) = (-2)^2 e^{-2x}$$
$$f^{(3)}(x) = (-2)^3 e^{-2x}$$
$$f^{(4)}(x) = (-2)^4 e^{-2y}$$
$$\vdots$$
$$f^{(n)}(x) = (-2)^n e^{-2x} \qquad \therefore f^{(n)}(0) = (-2)^n$$

Question 3. *(5 marks)* Evaluate the indefinite integral as an infinite series.

$$\int \frac{e^{-2x} - 1}{x}\, dx = \int \frac{\sum_{n=0}^{\infty} \frac{(-2)^n x^n}{n!} - 1}{x}\, dx$$

$$= \int \frac{\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n!}}{x}\, dx$$

$$= \int \sum_{n=1}^{\infty} \frac{(-2)^n x^{n-1}}{n!}\, dx = \sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n \cdot n!} + C$$

Bonus Question.[1] *(3 marks)* The Cantor ternary set $\mathscr{C}$ is created by iteratively deleting the open interval middle third from a set of line segments. One starts by deleting the open middle third $\left(\frac{1}{3}, \frac{2}{3}\right)$ from the interval $[0,1]$, leaving two line segments: $\left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$. Next, the open middle third of each of these remaining segments is deleted, leaving four line segments: $\left[0, \frac{1}{9}\right] \cup \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \frac{7}{9}\right] \cup \left[\frac{8}{9}, 1\right]$. The Cantor ternary set contains all points in the interval $[0,1]$ that are not deleted at any step in this ad infinitum. Compute the length of the deleted intervals.

[1] `https://en.wikipedia.org/wiki/Cantor_set`