**Dawson College**: **Linear Algebra and Vector Geometry**: 201-SN4-RE-S6: Winter 2026: **Quiz 8** name: ______________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.**

$$W = \left\{(x,y,z) \in \mathbb{R}^3 \;\middle|\; x+2y-z=0 \text{ and } 3x+5y+z=0\right\}.$$

a. *(1 mark)* Describe geometrically the set $W$ inside $\mathbb{R}^3$.

b. *(3 marks)* Find a basis for $W$.

c. *(1 mark)* State $\dim(W)$.

**Question 2.** *(5 marks)* Let

$$\mathbf{v}_1 = (1,2,-1), \qquad \mathbf{v}_2 = (2,3,1), \qquad \mathbf{w} = (a,b,c) \in \mathbb{R}^3.$$

Find necessary and sufficient conditions on $a,b,c$ so that $\mathbf{w} \in \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2\}$. When the condition is satisfied, express $\mathbf{w}$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

**Question 3.** *(5 marks)* Consider the set

$$S = \{(1,2,0),\ (2,5,1),\ (3,7,1)\} \subset \mathbb{R}^3.$$

a. *(3 marks)* Determine whether $S$ is linearly independent.

b. *(1 mark)* Determine whether $S$ spans $\mathbb{R}^3$.

c. *(1 mark)* Is $S$ a basis for $\mathbb{R}^3$?

**Question 4.** Let

$$B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\} = \{(1,0,1),\ (1,1,0),\ (0,1,1)\}.$$

a. *(3 marks)* Prove that $B$ is a basis for $\mathbb{R}^3$.

b. *(2 marks)* Find the coordinate vector $[\mathbf{x}]_B$ of $\mathbf{x} = (4,5,6)$.

c. *(2 mark)* Find the vector $\mathbf{y} \in \mathbb{R}^3$ such that

$$[\mathbf{y}]_B = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}.$$

**Question 5.** Let

$$\mathbf{u}_1 = (1,1,0), \quad \mathbf{u}_2 = (2,3,1), \quad \mathbf{u}_3 = (0,1,1), \quad \mathbf{u}_4 = (3,4,1).$$

a. *(4 marks)* Determine which vector(s), if any, are linear combinations of the preceding ones.

b. *(2 marks)* Extract a basis from the list $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.

c. *(1 mark)* State the dimension of the set of all linear combinations of these vectors.

**Question 6.** *(3 marks)* A student claims: *"If a set of three vectors in $\mathbb{R}^3$ spans $\mathbb{R}^3$, then each vector in the set must be nonzero, and no vector can be a linear combination of the other two."* Is the student correct? Give a careful justification.

**Bonus Question.** *(3 marks)* Prove that no vector subspace can have exactly two vectors